International Journal of Pattern Recognition
and Artificial Intelligence
Vol. 17, No. 6 (2003) 903–929

World Scientific
www.worldscientific.com

A METHODOLOGY FOR FEATURE SELECTION USING MULTIOBJECTIVE GENETIC ALGORITHMS FOR HANDWRITTEN DIGIT STRING RECOGNITION

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In this paper a methodology for feature selection for the handwritten digit string recognition is proposed. Its novelty lies in the use of a multiobjective genetic algorithm where sensitivity analysis and neural network are employed to allow the use of a representative database to evaluate fitness and the use of a validation database to identify the subsets of selected features that provide a good generalization. Some advantages of this approach include the ability to accommodate multiple criteria such as number of features and accuracy of the classifier, as well as the capacity to deal with huge databases in order to adequately represent the pattern recognition problem. Comprehensive experiments on the NIST SD19 demonstrate the feasibility of the proposed methodology.

1. Introduction

An important issue in constructing classifiers is the selection of the best discriminative features. In many applications, it is not unusual to find problems involving hundreds of features. However, it has been observed that beyond a certain point, the inclusion of additional features leads to a worse rather than better performance. Moreover, the choice of features to represent the patterns affects several aspects of the pattern recognition problem such as accuracy, required learning time, and the necessary number of samples.

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This apparent paradox presents us with a feature selection problem in automatic design of pattern classifiers. Such a problem refers to the task of identifying and selecting an effective subset of features to represent patterns from a larger set of often mutually redundant or even irrelevant features. This is not a trivial problem since features are seldom entirely independent. There may be redundancy, where certain features are correlated so that it is not necessary to include all of them in modeling, and interdependence, where two or more features between them convey important information that is obscure if any of them is included on its own.

In the context of practical applications such as handwriting recognition, feature selection presents a multicriterion optimization function, e.g. number of features and accuracy of classification. Genetic algorithms offer a particularly attractive approach to solve this kind of problems since they are generally quite effective in rapid global search of large, nonlinear and poorly understood spaces. It has been shown that genetic algorithms can be applied to general NP-complete problems. Moreover, simultaneous allocation of search effort to many regions of the search space contributes the power of genetic algorithm. It is called implicit parallelism and has been proved that it can set a lower bound of an N^3 speedup over systematic sequential search,[13] where N is the population size. In the last decade, genetic algorithms have been largely applied to the feature selection problem.[18,30,34,37] The approach often combines different optimization objectives into a single objective function. The main drawback of this kind of strategy lies in the difficulty of exploring different possibilities of trade-off between classification accuracy and different subsets of selected features. In order to overcome this kind of problem, Emmanouilidis *et al.*[6] proposed the use of a multiobjective genetic algorithm to perform feature selection.

In this work we discuss the use of multiobjective genetic algorithms as a means to search for subsets of features, which contain discriminatory information to classify handwritten digit strings. The proposed strategy takes into account an efficient multiobjective genetic algorithm[35] to generate a set of alternative solutions and the use of a cross-validation method to indicate the best accuracy/complexity (number of features) trade-off. We demonstrate that the cross-validation is very important when working with a set of alternative solutions and it cannot be neglected as in Ref. 6. The classification accuracy is supplied by multilayer perceptron neural networks in conjunction with the sensitivity analysis.[24] Such an approach makes it feasible to deal with huge databases in order to better represent the pattern recognition problem during the fitness evaluation. Some advantages of the proposed methodology include the ability to accommodate multiple criteria such as number of features and accuracy of the classifier, as well as the capacity to deal with huge databases in order to represent well the pattern recognition problem. Moreover, the use of the multiobjective genetic algorithms help to avoid the phenomenon of premature convergence presented by simple genetic algorithms. To make this paper more concise we have chosen to focus our attention on the proposed methodology rather than comparing it with classical approaches. This is based on the fact that several studies in the literature have been demonstrated that genetic algorithms

would be more effective than other methods when dealing with large-scale feature selection (i.e. more than 50 features). For those readers interested in comparative studies, please see Refs. 7, 17 and 34.

In order to show the robustness of the proposed strategy, we carry out experiments in two different contexts: isolated digits and string of digits. The latter is more difficult since we need to deal with problems such as fragmentation, overlapping and effects of segmentation. Finally, we analyze several solutions provided by the feature selection algorithm and show that there is a relationship between some discarded features and the strategy of zoning we choose. All experiments reported in this work use NIST SD19.

This paper is structured as follows. Section 2 summarizes different approaches to select feature subsets. Section 3 presents the general multiobjective problem as well as different strategies applied to solve such a problem. Section 4 describes the real world problem used in our experiments. Section 5 describes the proposed methodology and Sec. 6 reports the results of experiments carried out and some discussion as well. Finally, Sec. 7 presents some discussion and Sec. 8 concludes this paper.

2. Related Works

The preliminary works on feature selection started in the early 1960s. The approaches at that time were based on probabilistic measures of class separability and on entropies. In most of the methods the independence of features was assumed and the features were selected on the basis of their individual merits. Since such methods ignore the interactions among features, they usually produce unsatisfactory subsets of features.

In order to perform feature selection taking into account the relationship between features, three approaches can be found in the literature[3]: Complete, Heuristic and Randomized searches. Since complete search over all possible subsets of a feature set (2^N where N is the number of features) is not computationally feasible in practice, several authors have explored the use of heuristics for feature subset selection, often in conjunction with branch and bound search. Forward selection and backward elimination are the most common sequential branch and bound search algorithms used in feature selection.[14,25] Most of the current approaches assume monotonicity of some measure of classification performance. This ensures that adding features does not worsen the performance. However, many practical scenarios do not satisfy the monotonicity assumption. Moreover, this kind of search is not designed to handle multiple selection criteria.

Randomized algorithms make use of randomized or probabilistic steps or sampling processes. Several researchers have explored the use of such algorithms for feature selection[16,20] while others have explored the use of randomized population-based heuristic search techniques such as genetic algorithms for feature selection for decision tree and nearest neighbor classifier[15,31,34] or neural networks.[37,38] The

advantage of feature selection techniques that employ genetic algorithms is that they do not require the restrictive monotonicity assumption. They can also deal with the use of multiple selection criteria, e.g. classification accuracy, feature measurement cost, etc. This makes them particularly attractive in the design of pattern classifiers in many practical scenarios such as signature verification,[28] medical diagnosis,[37] facial modeling[12] and handwriting recognition.[15]

Due to the ability of genetic algorithms to deal with multiobjective optimization, several authors have explored genetic algorithms for feature selection for handwritten character recognition.[15,21,33] Feature selection using genetic algorithm is often performed by aggregating different objectives into a single and parameterized objective, which is achieved through a linear combination of the objectives. The main drawback of this approach is that it is very difficult to explore different trade-offs between accuracy and different subsets of selected features.

In order to overcome this kind of problem, Emmanouilidis *et al.*[6] proposed the use of a multicriteria genetic algorithm to perform feature selection. Notwithstanding that only small databases were considered in this work, they achieved interesting results.

2.1. *Filter and wrapper approaches to feature selection*

Feature selection algorithms can also be classified into two categories based on whether or not feature selection is performed independently of the learning algorithm used to construct the classifier. If feature selection is done independently of the learning algorithm, the technique is said to follow a filter approach. Otherwise, it is said to follow a wrapper approach.[14] While the filter approach is generally computationally more efficient than the wrapper approach, its major drawback is that an optimal selection of features may not be independent of the inductive and representational biases of the learning algorithm that is used to construct the classifier. On the other hand, the wrapper approach involves the computational overhead of evaluating candidate feature subsets by executing a given learning algorithm on the database using each feature subset under consideration.

3. Multiobjective Optimization Using Genetic Algorithms

3.1. *Definitions*

A general multiobjective optimization problem consists of a number of objectives and it is associated with a number of inequality and equality constraints. Mathematically, the problem can be written as follows.[29]

$$\begin{aligned} &\text{Minimize (or Maximize)} \quad f_i(x) \quad i = 1, \ldots, N \\ &\text{subject to :} \begin{cases} g_j(x) & \leq & 0 & j = 1, 2, \ldots, J \\ h_k(x) & = & 0 & k = 1, 2, \ldots, K\,. \end{cases} \end{aligned} \tag{1}$$

The parameter x is a p-dimensional vector having p decision variables. Solutions to a multiobjective optimization problem can be expressed mathematically in terms of nondominated or superior points. In a minimization problem, a vector $x^{(1)}$ is partially less than another vector $x^{(2)}$, $(x^{(1)} \prec x^{(2)})$, when no value of $x^{(2)}$ is less than $x^{(1)}$ and at least one value of $x^{(2)}$ is strictly greater than $x^{(1)}$. If $x^{(1)}$ is partially less than $x^{(2)}$, we say that the solution $x^{(1)}$ *dominates* $x^{(2)}$. Any member of such vectors which is not dominated by any other member is said to be *nondominated*. The optimal solutions to a multiobjective optimization problem are nondominated solutions. They are also known as Pareto-optimal solutions.

For example, in the case of minimization for two criteria,

$$\begin{cases} \text{Minimize} & f(x) = (f_1(x),\, f_2(x)) \\ \text{such that} & x \in X(\text{the feasible region}) \end{cases}$$

a potential solution $x^{(1)}$ is said to dominate $x^{(2)}$ iff:

$$\begin{aligned} &\forall i \ \in \{1,2\} : f_i(x^{(1)}) \leq f_i(x^{(2)}) \quad \wedge \\ &\exists j \ \in \{1,2\} : f_j(x^{(1)}) < f_j(x^{(2)})\,. \end{aligned} \tag{2}$$

3.2. *Classical approach*

A common difficulty with multiobjective optimization problem is the conflict between the objectives: in general, none of the feasible solutions allow simultaneous optimal solutions for all objectives. In other words, individual optimal solutions of each objective are usually different. Thus, mathematically the most favorable Pareto-optimum is that solution which offers the least objective conflict. One of the most classical methods is the weighted sum. In this strategy, multiple objective functions are combined into one overall objective $F(x)$ such that:

$$F(x) = \sum_{i=1}^{N} \omega_i f_i(x) \tag{3}$$

where $x \in X$, X is the objective space, the weights ω_i are fractional numbers $(0 \leq \omega_i \leq 1)$, and $\sum_{i=1}^{N} \omega_i = 1$. However, setting up an appropriate weight vector also depends on the scaling of each objective function. It is likely that different objectives take different orders of magnitude. When such objectives are weighted to form a composite objective function, it would be better to scale them appropriately so that each has more or less the same order of magnitude.

In this method, the optimal solution is controlled by the weight vector ω. It is clear from the above equation that the preference of an objective can be changed by modifying the corresponding weight. In most cases, each objective is first optimized and all objective function values are computed at each individual optimum solution. Afterwards, depending on the importance of objectives, a suitable weight vector is chosen and the single-objective problem given in Eq. (3) is used to find the desired solution.

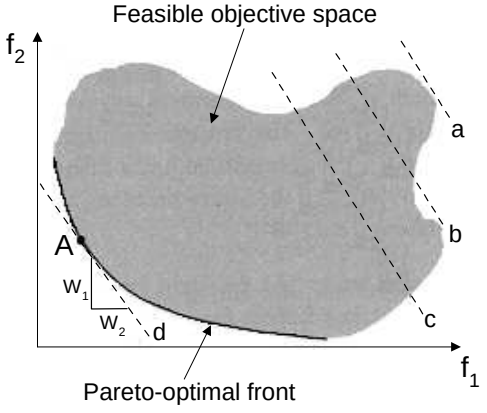


Fig. 1. Illustration of the weighted sum approach on a convex Pareto-optimal front.

Figure 1 shows an example of this approach where a two-objective problem is considered.[4] Once the weight vector is defined we can calculate the composite function F. Its contour surface can then be visualized in the objective space, as shown by lines "a", "b", "c" and "d". Since F is a linear combination of both objectives f_1 and f_2, we would expect a straight line as the contour line of F on the objective space. This is because any solution on the contour line will have the same F value. If considered carefully, this contour line is not an arbitrary one. Its slope is related to the choice of the weight vector. In fact, for two objectives, its slope is $-w_1/w_2$. The location of the line depends on the value of F on any point of the line. The effect of lowering the contour line from "a" to "b" is in effect jumping from solutions of higher F values to a lower one.

In a minimization problem, the task is to find the contour line with the minimum F value. This happens with the contour line which is tangential to the search space and also lies in the bottom-left corner of this space. In Fig. 1, this line is represented by "d". The tangent point "A" is the minimum solution of F, and is consequently the Pareto-optimal solution corresponding to the weight vector.

The only advantage of using this technique is the emphasis of one objective over the other. As we have seen, the optimization of the single objective may guarantee a Pareto-optimal solution but results in a single point solution. However, in real world situations we usually need different alternatives in order to make a decision. Moreover, if some of the objectives are noisy or have a discontinuous variable space, this method may not work properly. The main drawback of this approach is its sensitivity towards weights, i.e. the solution obtained largely depends on the underlying weight vector.

Deb in Ref. 4 mentioned further potential problems with these approaches, i.e. application areas where their use is restricted. Moreover, this kind of method requires several optimization runs to obtain an approximation of the Pareto-optimal set. As the runs are performed independently from each other, usually synergies cannot be exploited which, in turn, may create a high computation overhead.

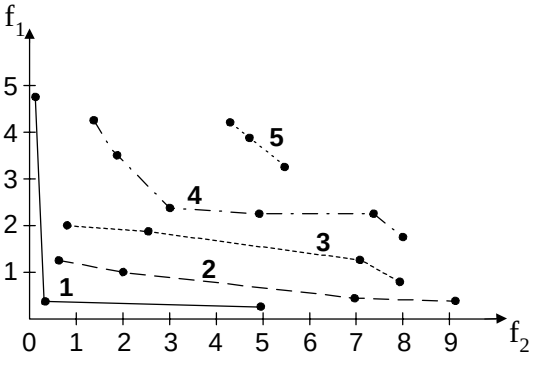


Fig. 2. Ranking of a population by fronts.

3.3. *Pareto-based approach*

In order to overcome the difficulties mentioned above, Pareto-based evolutionary optimization has become an alternative to classical techniques such as weighted sum method. This approach was first proposed by Goldberg in Ref. 9 and it explicitly uses Pareto dominance to determine the reproduction probability of each individual. Basically, it consists of assigning rank 1 to the nondominated individuals and removing them from contention, then finding a new set of nondominated individuals, ranked 2, and so forth. Figure 2 depicts the ranking by fronts.

Pareto-based ranking correctly assigns all nondominated individuals the same fitness, however, this does not guarantee that the Pareto set is uniformly sampled. When presented with multiple equivalent optima, finite populations tend to converge to only one of them, due to stochastic errors in the selection process. This phenomenon, known as genetic drift, has been observed in natural as well as in artificial evolution, and can also occur in Pareto-based evolutionary optimization.

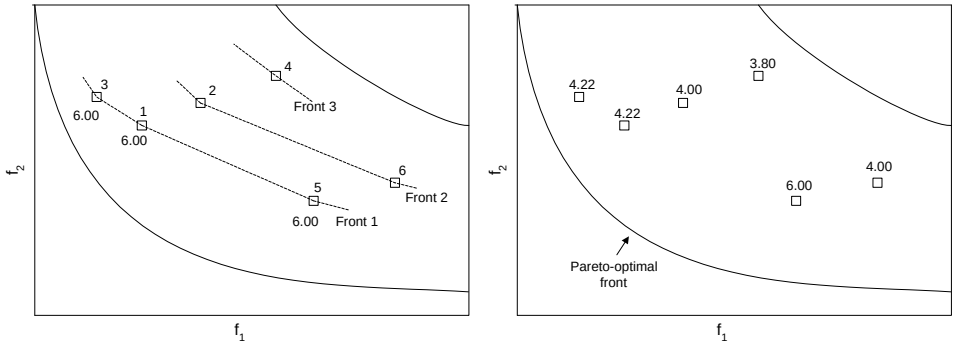
In order to avoid such a problem, Goldberg and Richardson in Ref. 10 proposed the additional use of fitness sharing. The main idea behind this is that individuals in a particular niche have to share the available resources. The more individuals are located in the neighborhood of a certain individual, the more its fitness value is degraded. In the following section we present the Pareto-based algorithm we have used as well as an implementation of the sharing function.

3.3.1. *Nondominated sorting genetic algorithm (NSGA)*

Over the past decade, a number of multiobjective evolutionary algorithms have been proposed. Zitzler *et al.* in Ref. 39 provided a systematic comparison of various evolutionary approaches to multiobjective optimization using six carefully chosen test functions. In this work, they found that NSGA (with elitism) proposed by Srinivas and Deb in Ref. 35 surpasses several other methods. Besides, such a method has been applied to solve various problems.[23,36] For these reasons, we opted to use such an algorithm in our study.

The idea behind the NSGA is that a ranking selection method is used to emphasize good points and a niche method is used to maintain stable subpopulations of good points. It differs from simple genetic algorithm only in the way the selection operator works. The crossover and mutation remain as usual. Before the selection is performed, the population is ranked based on an individual's nondomination. The nondominated individuals present in the population are first identified from the current population. Then, all these individuals are assumed to constitute the first nondominated front in the population and assigned a large dummy fitness value. The same fitness value is assigned to give an equal reproductive potential to all these nondominated individuals.

In order to maintain the diversity in the population, these classified individuals are then shared with their dummy fitness values. Sharing is achieved by performing selection operation using degraded fitness values obtained by dividing the original fitness value of an individual by a quantity proportional to the number of individuals around it. After sharing, these nondominated individuals are ignored temporarily to process the remaining population in the same way to identify individuals for the second nondominated front. These new sets of points are then assigned a new dummy fitness which is kept smaller than the minimum shared dummy fitness of the previous front. This process is continued until the entire population is classified into several fronts. It is illustrated in Fig. 3. We can observe from this figure that the individuals "1" and "3" had their fitness shared because they are close to each other. In this case, their fitness was reduced from 6.00 to 4.22. Then, the dummy fitness is assigned to the individuals of the second front by multiplying the lowest value of the first front by a constant k (let us say $k = 0.95$ for this example). Therefore, the individuals of the second front will receive a dummy fitness of 4.00 (4.22×0.95). Since the two individuals of the second front are not close to each other, their dummy fitness is maintained and a dummy fitness is assigned to the individual of the last front ($4.00 \times 0.95 = 3.80$).



(a) (b)

Fig. 3. Sorting population: (a) The population is classified into three nondominated fronts and (b) shared fitness values of six solutions.

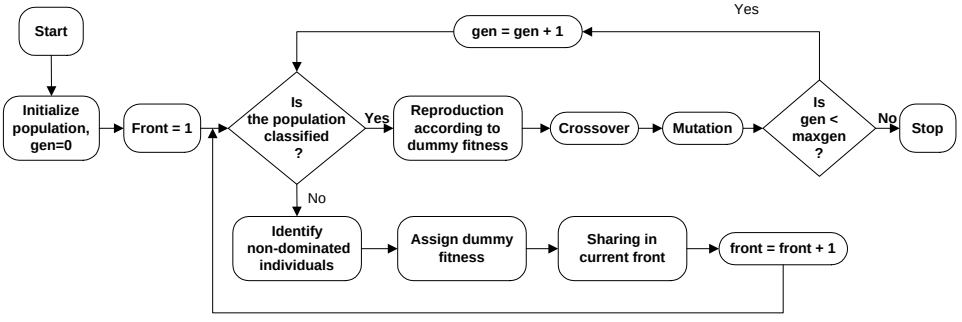


Fig. 4. Flow chart of NSGA.

The population is then reproduced according to the dummy fitness values. Since individuals in the first front have the maximum fitness value, they get more copies than the rest of the population. This was intended to search for the nondominated regions of Pareto-optimal fronts. The efficiency of NSGA lies in the way multiple objectives are reduced to a dummy fitness function using nondominated sorting procedures.

Figure 4 shows a flow chart of NSGA. The algorithm is similar to a simple genetic algorithm except for the classification of nondominated fronts and the sharing operation. The sharing in each front is achieved by calculating a sharing function value between two individuals in the same front as:

$$Sh(d(i,j)) = \begin{cases} 1 - \left(\dfrac{d(i,j)}{\sigma_{\text{share}}}\right)^2 & \text{if} \quad d(i,j) < \sigma_{\text{share}} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where $d(i,j)$ is the distance between two individuals i and j in the current front and σ_{share} is the maximum distance allowed between any two individuals to become members of a niche. In any application of sharing, we can implement either genotypic sharing, since we always have a genotype (the encoding), or phenotypic sharing. However, Deb and Goldberg in Ref. 5 indicated that in general, phenotypic sharing is superior to genotypic sharing. Thus, we have used a phenotypic sharing which is calculated from the normalized Euclidean distance between the objective functions.

The parameter σ_{share} can be calculated as follows[5]:

$$\sigma_{\text{share}} \approx \frac{0.5}{\sqrt[p]{q}} \tag{5}$$

where q is the desired number of distinct Pareto-optimal solutions and p is the number of decision variables. Although the calculation of σ_{share} depends on this parameter q, it has been shown in Ref. 35 that the use of the above equation with $q \approx 10$ works in many test problems.

4. Handwritten Digit String Recognition System

In order to evaluate the effect of the proposed feature selection scheme, we have used a handwritten string digit recognition system. Basically, the recognition module of this system is composed of one general-purpose recognizer, which is devoted to recognize ten numerical classes, and two verifiers devoted to detect over- and under-segmentation. All classifiers are neural networks (multilayer perceptron) trained with the backpropagation algorithm.[32] Figure 5 shows the block diagram of the system used in this work. The database considered in our work is the NIST SD19.

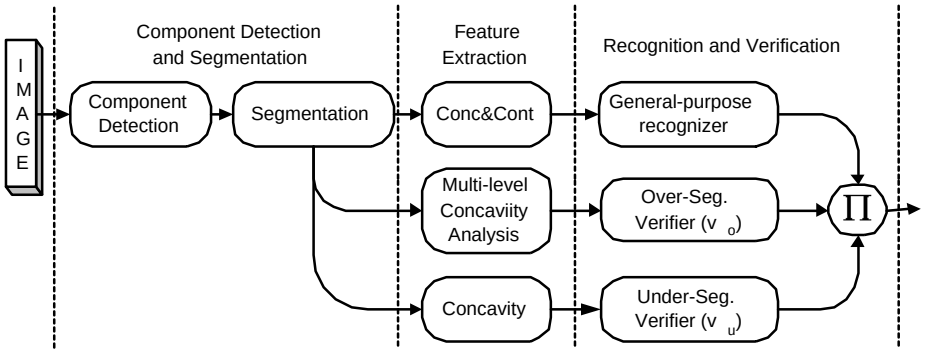


Fig. 5. Block diagram of the handwritten digit string recognition system.

Regarding the general-purpose recognizer, the training and validation sets contained 195,000 and 28,000 samples from the hsf_{0,1,2,3} series respectively while the test set contained 30,089 samples from the hsf_7 series. The recognition rates (zero-rejection level) achieved by the general-purpose recognizer were 99.66%, 99.45% and 99.13% on the training, validation and test sets, respectively. The feature vector used by this classifier is described in Sec. 4.1.

The over-segmentation verifier v_o has two outputs: isolated characters and over-segmentation. In order to train this verifier, we have used the following data: 8000 correctly segmented characters, 8000 naturally isolated characters and 12,000 over-segmented parts, which were generated automatically by the segmentation algorithm through the segmentation of the isolated and touching characters. The first two parts are devoted to train the first class of the verifier, while the third one is devoted to train the second class. Therefore, the training set used by v_o has 28,000 samples. The validation and test sets were built in the same manner, and they have 14,000 samples each. This verifier reached a recognition rate of 99.40% on the test set.

The under-segmentation verifier v_u is devoted to detect under-segmented characters. Thus, it considers two classes: isolated characters and under-segmentation. The database used in this case contains 9000 samples, which are divided into 5000 images of isolated characters and 4000 images of touching characters. The validation and test sets were built considering the same distribution of samples and they have

4000 samples each. This verifier reached a recognition rate of 99.17% on the test set.

The feature vector that feeds the first verifier is based on different levels of concavity while the second one is based on the same measurements of concavities used by the general-purpose recognizer. The difference lies in the kind of zoning used. Both feature vectors have 42 components. More details about this system can be found in Ref. 27.

4.1. *Feature set*

Since the focus of most of our experiments is the optimization of the feature set used by the general-purpose recognizer, we will describe it in this section. Such a feature vector is composed of a mixture of concavity and contour information. The basic idea of concavity measurements is the following: for each white pixel in the component, we search in 4-Freeman directions [Fig. 6(d)], the number of black pixels that it can reach as well as which directions the black pixel is not reached. When black pixels are reached in all directions [e.g. point x_1 in Fig. 6(b)], we branch out in four auxiliary directions [s_1 to s_4 in Fig. 6(c)] in order to confirm if the current white pixel is really inside a closed contour. Those pixels that reach just one black pixel are discarded.

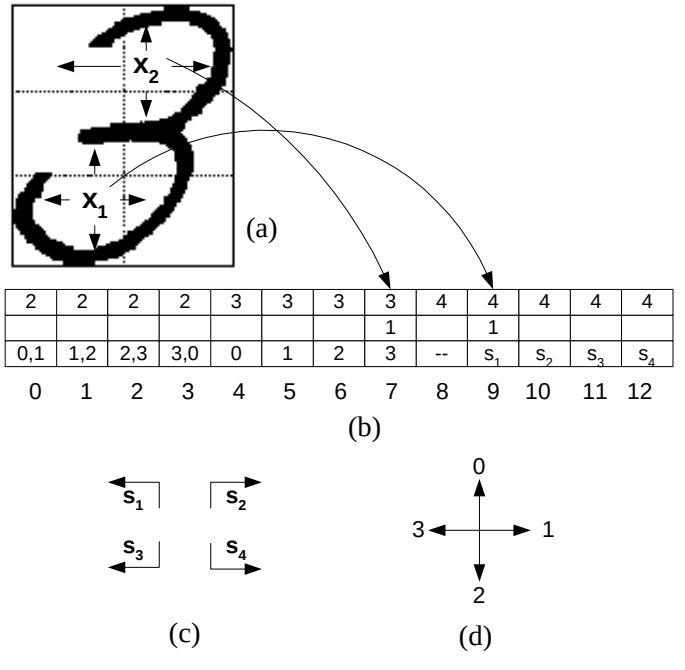


Fig. 6. Concavity measurements: (a) feature vector, (b) concavities, (c) auxiliary directions and (d) 4-Freeman directions.

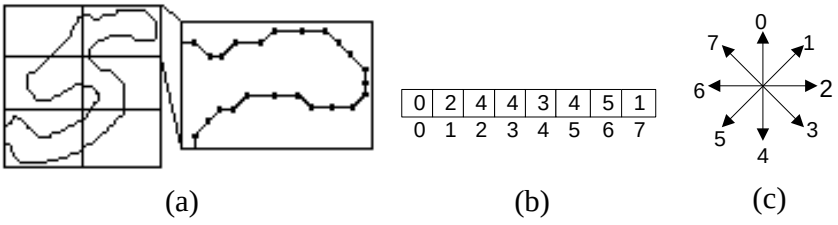


Fig. 7. Contour measurements: (a) contour image of the upper right corner zone, (b) feature vector and (c) 8-Freeman directions.

Thereafter, we increment the position in the feature vector that fits with results returned by the search [Figs. 6(a) and 6(b)]. In Fig. 6(a), we represent the feature vector where each component has two labels. The superior label means the number of black pixels found during the search while the inferior label means the directions where the black pixels were not reached. For example, the pixel x_2 [Fig. 6(b)] reaches the black pixel in all directions except in direction 3. Therefore, the position 7 of the feature vector is incremented. For pixel x_1, the position 9 is incremented because it reaches the black pixel in four directions. However, using the auxiliary direction s_1, we confirm that it is not inside a closed contour. When the pixel is inside a closed contour, the position incremented is the eighth.

Since we are dividing the image into six zones, we consider six feature vectors with 13 components each. Therefore, in the example presented above, the pixel x_2 will update the second vector while the pixel x_1 will update the fifth vector. Finally, the overall concavity feature vector is composed of (13×6) 78 components which are normalized between 0 and 1 by summing up their values and then dividing each one by this summation.

The contour information is extracted from a histogram of contour directions. For each zone, the contour line segments between neighboring pixels are grouped regarding 8-Freeman directions [Fig. 7(c)]. The number of line segments of each orientation is counted [Fig. 7(b)]. Therefore, the contour feature vector is composed of (8×6) 48 components normalized between 0 and 1. Finally, the last part of the feature vector is related to the surface of the character. We simply count the number of black pixels in each zone and normalize these values between 0 and 1. Thus, the final feature vector, which feeds e_{10} and e_3 has $(78 + 48 + 6)$ 132 components.

5. Methodology

Basically the proposed methodology works as follows: NSGA produces a set of solutions, called Pareto-optimal, which are validated through a second validation set not used during the learning of the classifier. Based on the validation curve, we pick one solution and thereafter train it to obtain a new optimized classifier. In the following subsection, we describe in detail the proposed methodology, which is depicted in Fig. 8.

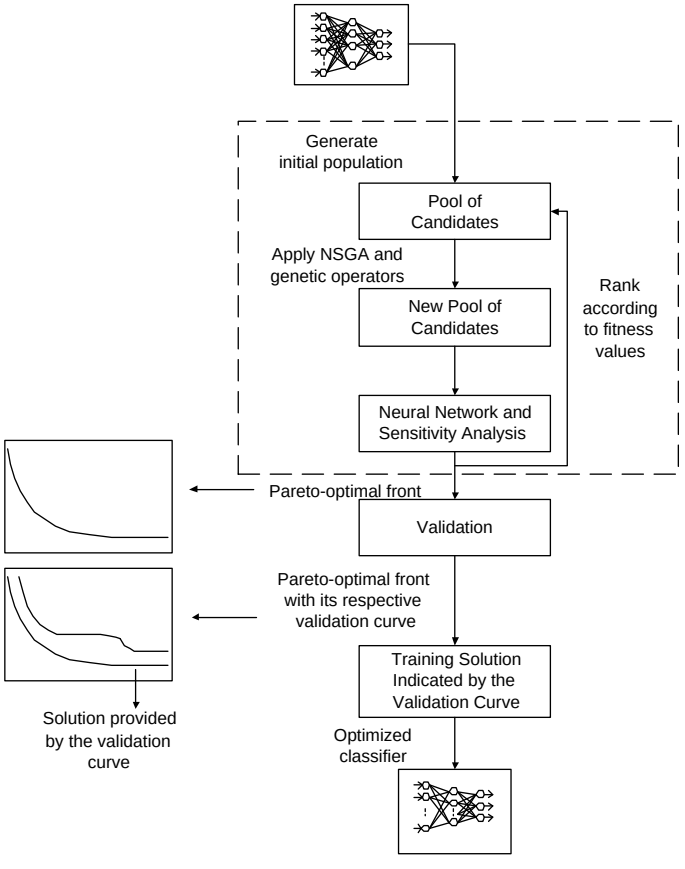


Fig. 8. Flow chart of the proposed methodology.

5.1. *Implementation of NSGA*

In our experiments, NSGA is based on bit representation, one-point crossover, bit-flip mutation, roulette wheel selection, and elitism which is implemented using a generational procedure. The following parameter settings were employed:

- Population size: 128.
- Number of generations: 1000.
- Probability of crossover (P_c): 0.8.
- Probability of mutation (P_m): 0.007.
- Niche Distance (σ_{share}): 0.45.

In order to define the probabilities of crossover and mutation, we have used the one-max problem, which is probably the most frequently-used test function in research on genetic algorithms because of its simplicity.[1] This function measures the fitness of an individual as the number of bits set to one on the chromosome.

We have used a standard genetic algorithm with a single-point crossover and the maximum generations of 1000. The fixed crossover and mutation rates are used in a run, and the combination of the crossover rates 0.0, 0.4, 0.6, 0.8 and 1.0 and the mutation rates of $0.1/L$, $1/L$ and $10/L$, where L is the length of the chromosome. The best results were achieved with $P_c = 0.8$ and $P_m = 1/L$. Such results confirmed the values reported by Miki *et al.*[22] The population size was determined through experimentation. The parameter σ_{share} was first defined using Eq. (5), and then tuned empirically.

As discussed elsewhere, our goal is to find the best accuracy/complexity trade-off for the classifier. This means that two objectives must be considered: minimization of the number of features and minimization of the error rate of the classifier. Computing the first one is simple, i.e. the number of selected features (bit = 1). The problem lies in computing the second one, i.e. the error rate supplied by the classifier. Regarding a wrapper approach, in each generation, evaluation of a chromosome (a feature subset) requires training the corresponding neural network and computing its accuracy. This evaluation has to be performed for each of the chromosomes in the population. Since such a strategy is not feasible due to the limits imposed by the learning time of the huge training set considered in this work, we have adopted the strategy proposed by Moody and Utans,[24] who used the sensitivity of the network to estimate the relationship between the input features and the network performance.

The sensitivity of the network model to variable i is defined as:

$$S_i = \frac{1}{N} \sum_j S_{ij} \tag{6}$$

where S_{ij} is the sensitivity computed for exemplar x_j

$$S_{ij} = SE(\bar{x}_i) - SE(x_{ij}) \tag{7}$$

with

$$\bar{x}_i = \frac{1}{N} \sum_{j=1}^{N} x_{ij} \,. \tag{8}$$

S_i measures the effect on the training squared error (SE) of replacing the ith input x_i by its average $\bar{x}_i$ for all N exemplars. Moody and Utans showed that when variables with small sensitivity values (S_i) with respect to the network outputs are removed, they do not influence the final classification. In light of this, to evaluate a given feature subset we replace all unselected inputs x_i of the test set by their respective averages $\bar{x}_i$ computed on the training set. In this way, we avoid training the neural network and hence turn the wrapper approach feasible for our problem. We call this strategy modified-wrapper. Such scheme was also employed by Yuan *et al.*[38]

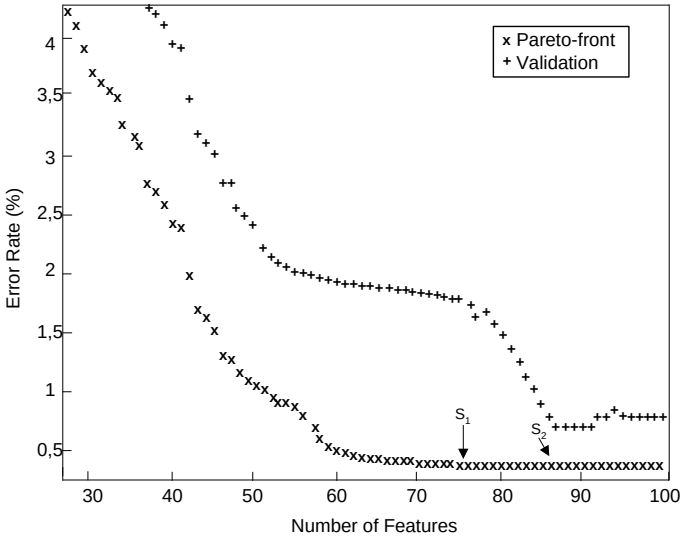


Fig. 9. Example of the Pareto-optimal front and its respective validation curve.

5.2. *Validating the Pareto-optimal front*

As depicted in Fig. 8, the last step of the strategy consists of choosing the best solution from the Pareto-optimal front. After several experiments, we realized that the Pareto-optimal front by itself does not provide enough information to select the best solution. Often, the best solution found in the Pareto-optimal front does not have good generalization power on a different database. In order to overcome this kind of problem, we propose the use of a validation database, which is not used during the feature selection procedure, to verify the generalization power of the Pareto-optimal front. This scheme is exemplified in Fig. 9.

If we analyze just the Pareto-front, the best trade-off between the number of features and the error rate is the solution S_1, which produces an error rate of 0.36% for 75 selected features. However, by analyzing the validation curve, we can observe that such a solution supplies a poor generalization on an unknown database and hence an error rate of about 1.7%. We can also observe that the accuracy/complexity trade-off that has the best generalization on the validation set is the solution S_2.

5.3. *Parallel processing*

The approach of parallel processing used in this work is based on a single-population master-slave genetic algorithm. In this strategy, one master node executes the genetic operators (selection, crossover and mutation), and the evaluation of fitness is distributed among several slave processors. In order to execute our experiments, we have used a Beowulf cluster with 17 (one master and 16 slaves) PCs (1.1Ghz

CPU, 512Mb RAM). Such a strategy, which is a low cost parallel computing based on a cluster of personal computers, makes the proposed approach tractable on short term.

In order to execute 1000 generations of NSGA, this architecture takes about 8 hours. The same process in a SUN Ultra 1 (167Mhz CPU, with 128Mb RAM) takes more than 20 days.

6. Experiments and Results

In order to test the effectiveness of the proposed methodology we have applied it to optimize the system described in Sec. 4. As mentioned before, we have performed experiments in two different contexts: isolated digits and strings of digits. For all reported results we used the following definitions of the recognition rate, error rate, rejection rate and reliability rate. Let B be a test set with N_B string images. If the recognition system rejects N_{rej}, classifies correctly N_{rec} and misclassifies the remaining N_{err}, then

$$\text{Recognition Rate} = \frac{N_{\text{rec}}}{N_B} \times 100 \tag{9}$$

$$\text{Error Rate} = \frac{N_{\text{err}}}{N_B} \times 100 \tag{10}$$

$$\text{Rejection Rate} = \frac{N_{\text{rej}}}{N_B} \times 100 \tag{11}$$

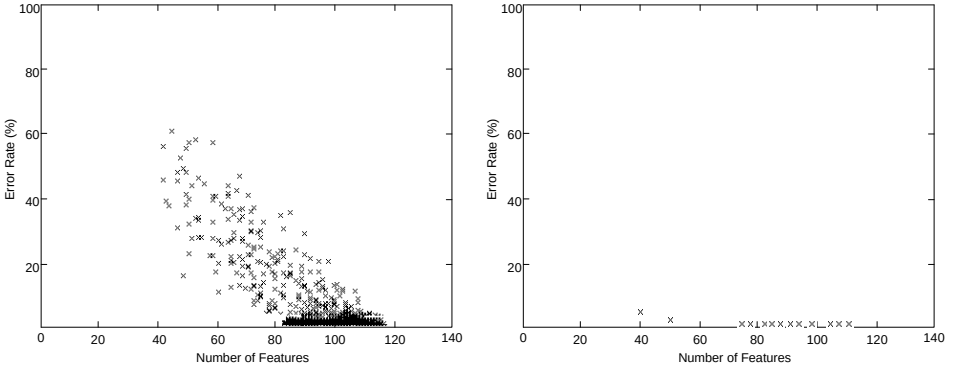
$$\text{Reliability} = \frac{N_{\text{rec}}}{N_{\text{rec}} + N_{\text{err}}}\,. \tag{12}$$

Therefore, the recognition rate, error rate and rejection rate sum up to 100%. In general, the recognition rate is a valid rate, but to characterize the quality of the classifier for practical issues, this rate is not appropriate. A more suitable rate for classifier examination in real applications is the reliability rate, which gives an impression of the classifier behavior in several different situations. Such a rate takes not only the error rate, but also the reject rate into consideration. Therefore, for all experiments presented in this work we report the recognition rate at zero-rejection level, 0.5% error level and reliability rate. We have used the rejection mechanism proposed by Fumera *et al.*[8] In all experiments we do not consider the knowledge of the number of digits in the string.

6.1. *Experiments on isolated digits*

The database used to assess the impact of different subsets of inputs during the feature selection was the same as employed for validation during training of the classifier (28,000 samples from hsf_0123).

In order to show the limitations of the weighted-sum approach, we first tried to optimize the classifier using it. As expected, the results achieved by the weighted-sum approach presented a premature convergence to a specific region of the search



(a) (b)

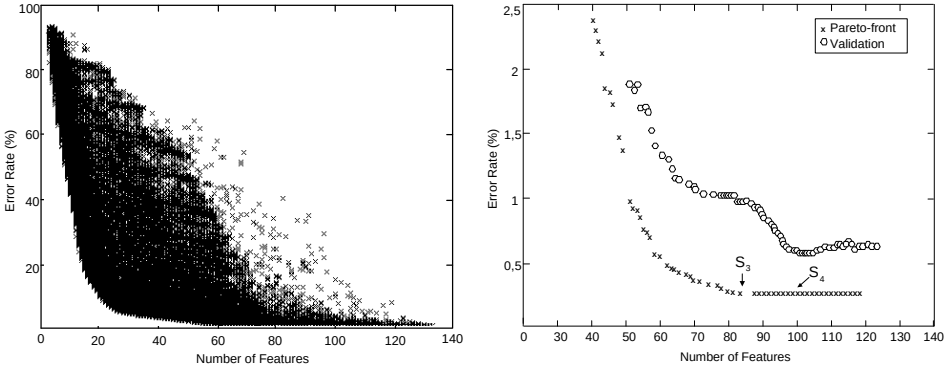
Fig. 10. Proposed methodology using the weighted-sum approach: (a) evolution of the population in the objectives plane (one trial) and (b) Pareto-optimal solutions found by the classical approach after several trials.

space instead of maintaining a diverse population. This kind of behavior can be explained by the sensitivity towards weight presented by the weighted-sum approach. Since we have chosen weights to favor solutions with a small error rate rather than a small number of features, the selection pressure drove the search to the region where the error rates are smaller. Thus, after several trials using different weights we did not succeed in finding the Pareto-optimal front but rather an approximation of the Pareto-optimal solutions (Fig. 10).

As we have discussed in Sec. 3.3, the Pareto-based approach was designed to overcome this kind of problem. Since NSGA uses a niching technique to preserve the diversity in the population, this algorithm is able to deal with the problem of converging prematurely to a specific region of the search space. Therefore, it can guide the search towards the Pareto-optimal set. Figure 11(a) depicts the evolution of the population in the objectives plane from the first generation to the last one. This plot demonstrates the efficacy of NSGA in converging close to the Pareto-optimal front with a wide variety of solutions.

As discussed in Sec. 5, after finding the Pareto-optimal front the next step is to find a solution. In order to perform this task we have used a new validation database, which consists of 30,000 samples (from hsf_7) not used so far. Figure 11(b) shows the Pareto-optimal front as well as its correspondent validation curve, which depicts the performance of the entire Pareto-optimal front on this new validation set. After analyzing the validation curve plotted in Fig. 11(b), we selected a solution with 100 features [solution S_4 in Fig. 11(b)] and error rate on the new validation set smaller than 1% to retrain the general-purpose recognizer.

Thereafter, we trained a new classifier using such a solution using the same databases presented in Sec. 4. The recognition rates achieved by this new classifier at zero rejection level were 99.66%, 99.63%, 99.16% on training, validation and test sets, respectively. Table 1 summarizes the results obtained by the optimized



(a) (b)

Fig. 11. Feature selection using a Pareto-based approach: (a) evolution of the population in the objective plane, (b) Pareto-optimal front found by NSGA and its correspondent validation curve.

Table 1. Comparison between the original and optimized classifiers. Error rate fixed at 0.5%.

Original System				Optimized System			
Number of	Rates (%)			Number of	Rates (%)		
Features	Recog.	Rej.	Reliab.	Features	Recog.	Rej.	Reliab.
132	98.50	1.00	99.49	100	98.54	0.96	99.50

classifier on the test set at 0.5% error level. As we can observe, the optimized system uses about 25% less features and keeps the reliability rate at the same level. Such results confirm the efficiency of the proposed methodology in selecting a powerful subset of features. In order to show the importance of the validation set to select a good solution, we retrained the best trade-off of the Pareto-front without regarding the validation curve [solution S_3 in Fig. 11(b)]. The recognition rate (zero-rejection level) and reliability rate reached by this solution on the test set were 96.8% and 98.5%, respectively.

6.2. *Experiments on strings of digits (1)*

The experiments using numeral strings are based on 12,802 numeral strings extracted from the hsf_7 series and distributed into six classes: 2_digit (2370), 3_digit (2385) 4_digit (2345), 5_digit (2316), 6_digit (2316) and 10_digit (1217) strings, respectively. These data exhibit different problems such as touching and fragmentation. Figure 12 shows some examples of the variability found in the NIST database. The recognition rates at zero-rejection level as well as the performance at 0.5% error level achieved by the original system are reported in Table 2.

In the first experiment with strings of digits we just replaced the original classifier by the optimized one presented in the previous section. The idea here is to find out whether the optimized classifier is good enough to recognize strings of digits.

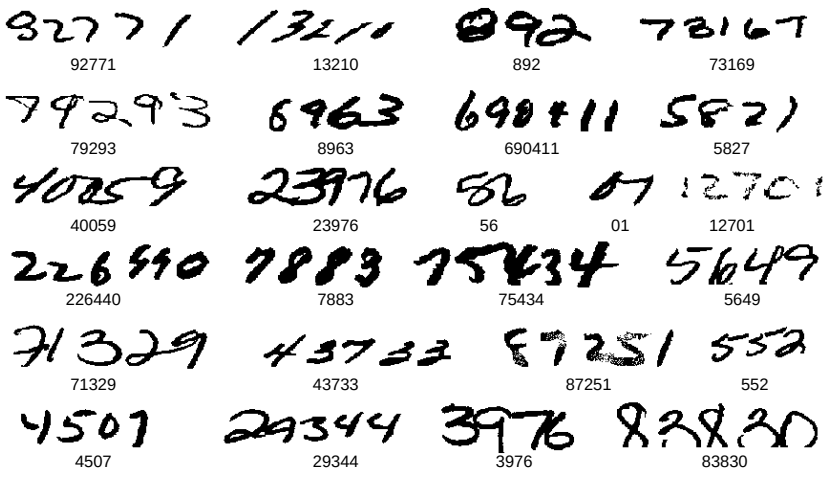


Fig. 12. Examples of digit strings (NIST SD19).

Table 2. Performance of the original system on strings of digits.

String Length	Number of Strings	Rec. Rate	Rec. Rate (Error = 0.5%)	Reliability Rate
2	2370	96.88	93.88	99.47
3	2385	95.38	89.84	99.44
4	2345	93.38	84.36	99.41
5	2316	92.40	82.44	99.40
6	2169	93.12	84.03	99.41
10	1217	90.24	75.20	99.34

As discussed before, such a problem is much more complicated than the problem of naturally isolated digits since the system must face complications such as noise, fragmentation and touching digits. In order to answer the above question, we have applied the optimized classifier to recognize strings of digits. It is worthy of remark that both verifiers remain the same. The results of these experiments are presented in Table 3.

Table 3. Performance of the system on strings of digits using the general-purpose classifier optimized with isolated digits.

String Length	Number of Strings	Rec. Rate	Rec. Rate (Error = 0.5%)	Reliability Rate
2	2370	97.21	94.00	99.47
3	2385	94.62	89.72	99.44
4	2345	93.34	84.38	99.41
5	2316	92.36	82.67	99.40
6	2169	92.60	83.70	99.40
10	1217	89.19	74.77	99.33

Table 4. Performance of the system on strings of digits using the general-purpose classifier optimized with strings of digits.

String Length	Number of Strings	Rec. Rate (Error = 0.0%)	Rec. Rate (Error = 0.5%)	Reliability Rate
2	2370	96.91	93.90	99.47
4	2345	93.33	84.36	99.41
5	2316	92.40	82.44	99.40
6	2169	92.35	83.95	99.40

At the second time, we carried out the feature selection for both verifiers and used them in the system afterwards. To perform this task, we have used the proposed methodology where the databases used to validate the Pareto-front (called second validation set, so far) for the over-segmentation and under-segmentation verifiers have 7000 and 2000 samples, respectively. They were built in the same way we have described in Sec. 4. After running the feature selection we found the solution with 30 and 38 features for the over-segmentation and under-segmentation verifiers, respectively.

After training these optimized verifiers, we applied them together with the optimized classifier to recognize the strings of digits. The reliability rates were kept at the same level as presented in Table 3. Considering the three optimized classifiers in this experiment, the total features used by the system were reduced from 216 (132+42+42) to 168 (100+30+38), i.e. about 22% less features.

In spite of the fact that the optimized classifier succeeded in keeping the reliability rate at the same levels as the original system, such a classifier produced better recognition rates for strings composed of naturally isolated digits and worse performance for strings that showed problems of touching digits, fragmentation and noise. Therefore, the improvement found in the former case compensates the problems found in the latter.

6.3. *Experiments on strings of digits (2)*

In order to improve the performance of the classifier globally, i.e. for naturally isolated digits as well as digits with problems of fragmentation, touching and so forth, we have performed a new series of experiments. The goal was to select a subset of features suitable for the problem of strings of digits rather than the problem of naturally isolated digits. Thus, instead of using a database of isolated digits to assess the fitness during the feature selection procedure, we used the 3-digit string database which is composed of 2385 images. In this manner, problems such as overlapping, fragmentation and effects of segmentation are tackled during the optimization process and hence relevant features for those problems will not be discarded. The validation set used to find the best solution in the Pareto-optimal front is the 10-digit string database which contains 1217 images.

After performing the feature selection for the general-purpose recognizer taking into account such databases, the best solution we found had 124 selected features,

i.e. it is considerably greater than that presented in the previous section (100 features). After training such a classifier and using it to classify strings of digits we performed an error analysis in order to find out whether the optimized classifier got better results for strings with problems of touching digits, fragmentation and noise. We verified that it succeeded in improving the rates for strings with such problems, however, the rates for strings composed of naturally isolated digits was slightly lower than those shown in the previous section. Since most of the strings of the NIST database are composed of naturally isolated digits (about 75%), the slightly worse rates obtained in this case neutralized the improvement reached for strings with problems. Hence, the reliability rates of this experiment are almost the same as those of the previous experiment. Table 4 reports the performance (0.5% error level) of the classifier optimized in the context of strings of digits. The results for 3- and 10-digit strings were omitted here since we have used such subsets during the optimization process.

7. Discussion

In spite of the fact that the Pareto-based approach presents several advantages over the classical one, we have seen through the experiments that both strategies found similar solutions [see Figs. 10(b) and 11(b)]. In our first experiment, we observed that the classical approach converged the search to the space where the most probable solutions are located due to the weights we have chosen. However, for problems where the solutions are located along the Pareto-front, the classical approach does not work properly. Moreover, to achieve part of the Pareto-front, the weighted-sum method was run several times with different weight vectors.

For the problem of feature selection for handwriting recognition we can observe that the main advantage of the Pareto-based approach is the ability of dealing with different databases without having to deal with problems such as scaling and finding the suitable values for the weight vector. Moreover, Pareto-based approaches have the ability of finding the Pareto-optimal front in the first run of the algorithm.

As we have seen, two different optimized classifiers were used to recognize strings of digits. The former was optimized considering a database composed of naturally isolated digits only while the latter took into account a database of strings of digits. Through detailed experiments we have demonstrated that in both cases the optimized systems attained very similar reliability rates. In the first case, the classifier obtains slightly better rates for isolated digits and worse rates for strings with problems such as touching digits, fragmentation and noise while in the second case we observed the opposite. Since about 75% of the strings of digits of the NIST database consist of naturally isolated digits, the classifier optimized in this context, which has about 25% less features than the original classifier, provided the same reliability rates as those found in the original system.

Table 5 summarizes the recognition rates claimed by different authors on NIST database. Ha *et al.*[11] used about 5000 strings of the NIST SD3 while Lee and

Table 5. Recognition rates on NIST databases reported by other authors.

Authors	String Length	Number of Tested Strings	Error Rate 0%	Error Rate 1%	Error Rate 0,5%
Ref. 11	2	981	96.20	93.50	91.50
	4	988	93.20	81.00	70.00
	5	988	91.10	77.50	70.50
	6	982	90.30	75.50	66.50
Ref. 19	2	1000	95.23	95.20	–
	4	1000	80.69	80.50	–
	5	1000	78.61	78.40	–
	6	1000	70.49	70.20	–

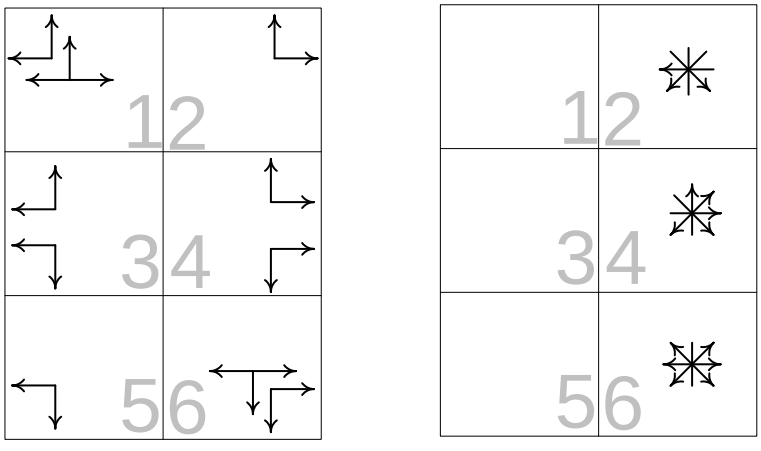
Kim[19] used 5000 strings but they did not specify the data used. By comparing the results reached by our system (Table 4) with those reported by other authors (Table 5), we can confirm that our system provides very good recognition rates at zero-recognition level and a very encouraging error-reject trade-off.

Since there is an interdependence between features where two or more features between them convey important information, it is very difficult to analyze the unselected features independently. However, analyzing the unselected features of several runs of the algorithm we could notice the following aspects.

1. In most of the solutions we observed that the unselected features of concavity have a certain symmetry among the zones, e.g. zones 1–6, 2–5 and 3–4 [Fig. 13(a)]. Moreover, we can conclude that for this kind of zoning there is a relationship between the geographic position of the zone and 4-Freeman directions. For example, in zone 2 (North–East) the unselected concavity configuration is the one that searches black pixels in directions North–East. We can notice the same behavior for the other zones.
2. All unselected features of contour are located just on the right side of the zoning [Fig. 13(b)] in most solutions. In this case we can assume that such features are correlated so that it is not necessary to include them in the feature set.
3. The information related to the surface of the image was never unselected. This means that such an information is relevant to the feature vector we have used.

It is worth to remark that such an analysis is valid for the feature vector described in Sec. 4.1. However, it can be a very helpful tool to get a better insight for any kind of feature vector.

Finally, to demonstrate that the methodology can be successfully extended to different features, we have applied it on two other feature sets, namely, Distances[26] and Edge Maps.[2] Thus, two classifiers were trained using the same databases and methodology described in Sec. 4. Table 6 reports the performance achieved by both classifiers on the test set of isolated digits (30,089 samples of hsf_7). It can be observed from this table that the optimized feature sets get slightly better results



(a) (b)

Fig. 13. Unselected features for handwritten digit recognition: (a) concavities and (b) contour.

Table 6. Results on two different features (Distances and Edge Maps).

	Original Classifiers				Optimized Classifiers			
Feature Set	No. Feat.	RR. (%)	RR. Err = 0.5%	Rej. (%)	No. Feat.	RR. (%)	RR. Err = 0.5%	Rej. (%)
Distances	96	98.17	92.80	6.70	90	98.21	93.05	6.45
Edge Maps	125	97.04	85.10	14.40	105	97.10	86.00	13.50

using less features for zero-rejection level and error rate fixed at 0.5% as well. Such a table can be read in the following way: the original feature set of Distances is composed of 96 components, reaches the recognition rate (RR) at zero-rejection level of 98.17%, and 92.80% at error rate fixed at 0.5% (rejecting 6.70%). After feature selection, it contains 90 components, reaches the recognition rate (RR) at zero-rejection level of 98.21%, and 93.05% at error rate fixed at 0.5% (rejecting 6.45%).

8. Conclusion

In this study we have proposed a methodology for feature selection which uses a Pareto-based approach to generate the Pareto-optimal front where sensitivity analysis and neural network enable the use of a representative database to evaluate fitness. Thereafter, the entire Pareto-front is validated on a different database in order to provide the solutions that present better generalization on different databases. Finally, the selected solution is trained and applied to the recognition system.

The use of a Pareto-based approach instead of a classical one is supported by the theory as well as the experiments carried out. We have also shown the importance of using a validation set in order to avoid selecting subsets of features with poor

generalization ability. Through comprehensive experiments on isolated digits and strings of digits, we have demonstrated that the proposed methodology succeeded in reducing the complexity of feature set used by the classifier and also that even using less features such a classifier achieved reliability rates at the same level as those reached by the original classifier.

We have seen that the classifier optimized in the context of isolated digits produced reliability rates for the problem of strings of digits at the same level as the classifier optimized in the context of strings of digits. However, we also pointed out that such a behavior is closely related to the database we have used, since it contains about 75% of naturally isolated digits. Finally, we have analyzed several solutions provided by the feature selection algorithm and determined the relationship between some unselected features and the strategy of zoning we have applied.

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