Image segmentation with a statistical shape prior

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October 26, 2015
Shape prior based image segmentation

Foulonneau'04
Etyngier'07
Prior information based segmentation

Our assumption: $n$ atlases or label maps

→ Atlas registration based approaches
→ Statistical shape prior based approaches

Lotjönen’10
Outline

• Related works in prior information segmentation
  – Atlas based approaches
  – Statistical shape prior based approaches

• Manifold learning for shape set modelling

• ML-based shape prior segmentation framework

• A few results on cardiac MRI
Multi-atlas registration for image segmentation

Kirisli’10
Multi-atlas: recent developments

Subject to be segmented

Subject selection

The N closest training subjects selected from the dataset
Multi-atlas: recent developments

Patch-based (non-local) approaches

\[ w(x, y) = f \left( \frac{1}{K} \sum_{x' \in P(x), y' \in P(x')} (I(x') - I(y'))^2 \right) \]

\[ \forall x \in \Omega, V(x) = \frac{\sum_{y \in N(x)} w(x, y) L(y)}{\sum_{y \in N(x)} w(x, y)} \]

Rousseau TMI’10
Coupé NeuroImage’11
Statistical shape model for image segmentation

• **Objective:**
  
  – learn the possible shape deformations of an object statistically from a set of training shapes
  
  – restrict the contour deformation to the subspace of familiar shapes during the segmentation process

  – Active Shape Models, Cootes 1995

  – Leventon CVPR’00, Tsai TMI’03

    • Implicit representation
Statistical shape model for image segmentation

Example: Tsai’s framework

Shapes are represented as signed distance functions

\[
\mathbb{D}_\gamma = \varepsilon(x) \inf_{y \in \partial s} d(x, y) \text{ with } \varepsilon(x) = \begin{cases} 
+1 & \text{if } x \in s, \\
-1 & \text{if } x \notin s
\end{cases}
\]

After rigid alignment:

\[
\Phi(w, p)(x, y) = \Phi(\tilde{x}, \tilde{y}) + \sum_{i=1}^{k} w_i \Phi_i(\tilde{x}, \tilde{y})
\]

Mean shape  Eigenshapes
Statistical shape model for image segmentation

Example: Tsai’s framework

\[
\Phi[w, p](x, y) = \bar{\Phi}(\bar{x}, \bar{y}) + \sum_{i=1}^{k} w_i \Phi_i(\bar{x}, \bar{y})
\]

\[
E_{cv} = \int_{R^u} (I - \mu)^2 dA + \int_{R^v} (I - \nu)^2 dA
\]

\[
w^{(t+1)} = w^{(t)} - \alpha_w \nabla w E
\]

\[
p^{(t+1)} = p^{(t)} - \alpha_p \nabla p E
\]
Problems of linear shape space

• Assumes the data lie in a linear subspace
• permissible shapes are assumed to form a multivariate Gaussian distribution

Yet: real world data sets present complex deformations

• Non linear shape statistics for image segmentation
  – introduced with kPCA in Cremers, ECCV’02
  – with manifold learning techniques: Etyngier’07, Yan’13, Moolan-Ferouze’14...
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Manifold learning

• process of recovering the **underlying low dimensional structure** of a manifold that is embedded in a higher-dimensional space

• closely related to the notion of dimensionality reduction

\[ \Xi = [\xi_i]_{1 \leq i \leq N} \]

\[ X = [x_i]_{1 \leq i \leq N} \]

Each shape SDM is vectorized

Source: J. Lee, UCLouvain, nov’14 seminar
Principle of spectral ML techniques

• Compute a similarity matrix $M \ (n \times n)$ between $n$ points (shapes for us) of the dataset
  – Goal: to connect points that lie within a common neighbourhood.
  
• $k$-nearest neighbour or $\epsilon$-ball

http://isomap.stanford.edu/
Principle of spectral ML techniques

- Compute a similarity (affinity) matrix $M$ \((n \times n)\)
- From $M$, compute a feature matrix $F$:
  - size $n \times n$
  - symmetric
  - positive semi definite
- Spectral decomposition of $F$
- Keep the $m$ smallest/largest eigenvectors

Cf Shi & Malik’s Normalized cuts (PAMI’00)
An example

- Number two in MNIST database (n=500)
  - Images: 20 x 20
  - d = 400 → m = 2
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How to use an non linear shape prior for segmentation?

- Iterative process:
  \[ S = \begin{pmatrix} s_1 & \ldots & s_n \end{pmatrix} \]
  \[ s_i \in \mathbb{R}^d \]

  - Energy-based image segmentation
  - Projection of the binary segmentation into the reduced space
  - Constrain the shape to resemble its neighbours and backprojection
  - Rigidly align the shape

  \[ x^* = \frac{1}{\lambda_k} \sum_{j=1}^{n} u_{kj} K(x^*, x_j) \]

  - Out-of-sampling:
    - Which dimension?
    - Which shape in the segmentation result?
    - Expresses the affinity between \( x^* \) and all points \( x_j \)

Based on Etyngier ICCV’07 & Moolan-Ferouze MICCAI’14
How to use an non linear shape prior for segmentation?

• **Iterative process:**

\[ S = \left( s_1 \ldots s_n \right) \]

\[ s_i \in \mathbb{R}^d \]

- Energy-based image segmentation
- Projection of the binary segmentation into the reduced space
- Rigidly align the shape
- Constrain the shape to resemble its neighbours and backprojection
- Manifold computed from a set of manually segmented images

\[ x^* : \text{shape of the segmentation result in the reduced space} \]

\[ X = \left( x_1 \ldots x_n \right) \]

\[ x_i \in \mathbb{R}^m \]

Based on Etyngier ICCV’07 & Moolan-Ferouze MICCAI’14
Constrain the shape in the embedding

- Find the shape nearest neighbors (NN)
- The shape \( \hat{s} \) is a linear combination of its NN:
  \[
  \hat{s} = \sum_{i=0}^{m} \theta_i s_i \quad \text{with} \quad \sum_{i=0}^{m} \theta_i = 1 \quad \text{and} \quad \theta_i \geq 0, \forall i = 0, \ldots, m
  \]

  \[
  \hat{\theta} = \arg\min d \ (s^*, \hat{s})
  \]

  with  
  \[
  d \ (s^*, \hat{s}) = \sum (H(s^*) - H(\hat{s}))^2
  \]
How to use an non linear shape prior for segmentation?

• **Iterative process:**

\[
S = \begin{pmatrix} s_1 & \ldots & s_n \end{pmatrix}, \quad s_i \in \mathbb{R}^d
\]

Energy-based image segmentation

Out-of-sample

Projection of the binary segmentation into the reduced space

x*: shape of the segmentation result in the reduced space

\[
X = \begin{pmatrix} x_1 & \ldots & x_n \end{pmatrix}, \quad x_i \in \mathbb{R}^m
\]

Manifold computed from a set of manually segmented images

Constrain the shape to resemble its neighbours and backprojection

Rigidly align the shape

\[\hat{S}\]

Take into account the constrained shape in an energy term

Based on Etyngier ICCV’07 & Moolan-Ferouze MICCAI’14
How to use an non linear shape prior for segmentation?

• Iterative process:

1. Energy-based image segmentation
2. Projection of the binary segmentation into the reduced space
3. Constrain the shape to resemble its neighbours and backprojection
4. Rigidly align the shape

Take into account the constrained shape in an energy term

\[ E_{total}(L) = E_{data}(L) + E_{smooth}(L) + E_{prior}(L) \]

Find the labeling \( L \) such that \( E(L) \) is minimum
Shape prior energy term

Find the labeling $L$ such that $E(L)$ is minimum

$$E_{\text{prior}}(O)$$ designed to be small for pixels likely to be labelled as object

$$E_{\text{prior}}(B)$$

From $\hat{s}$, we define a probability atlas

$$M(i) = \begin{cases} 
1 & \text{if } \hat{s}(i) \leq 0 \\
\frac{1}{e^{-\hat{s}(i)/\gamma_f(i)}} & \text{if } \hat{s}(i) > 0 
\end{cases}$$

- Binary shape
- Signed distance map
Shape prior energy term

Find the labeling $L$ such that $E(L)$ is minimum

$$E_{\text{prior}}(O) \quad \text{designed to be small for pixels likely to be labelled as object}$$

$$E_{\text{prior}}(B) \quad \text{background}$$

From $\hat{s}$, let's define a probability atlas

**Shape prior term:**

$$E_{\text{prior}}(O) = - \sum_i \log(M(i))$$

$$E_{\text{prior}}(B) = - \sum_i \log(1 - M(i))$$

Moolan-Ferouze MICCAI’14

$E_{\text{total}}$ is minimized with the mincut – maxflow algorithm [Boykov+Kolmogorov’04]
Experimental results

• Application: segmentation of the right ventricle in cardiac MRI

• Implementation:
  – Manifold learning: diffusion maps (Etyngier’07)
  – Graphcut based image segmentation
  – Shapes are described with signed distance maps
Experimental results

RV shape in 2D space

(intrinsic dim≈3)
Experimental results

Initializations

Final segmentations
Some perspectives with ML techniques

- Also investigated for atlas-based approaches
  - **ML for atlas selection** [Wolz NeuroImage’10, Cao MICCAI’11, Hoang-Duc PlosOne’13, Gao SPIE’14]

Cao MICCAI’11
Some perspectives with ML techniques

• Also investigated for atlas-based approaches
  • ML for atlas selection [Wolz NeuroImage’10, Cao MICCAI’11, Hoang-Duc PlosOne’13, Gao SPIE’14]
  • Patch-based approaches [Shi et al MICCAI’14, Oktay et al MICCAI’14]
    – Sparse representation and dictionary learning

Oktay et al MICCAI’14
Thank you...

• ... for your attention.
• Comments? Questions?

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