Random Forests
—
Parametrization and Dynamic Induction

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Random Forests

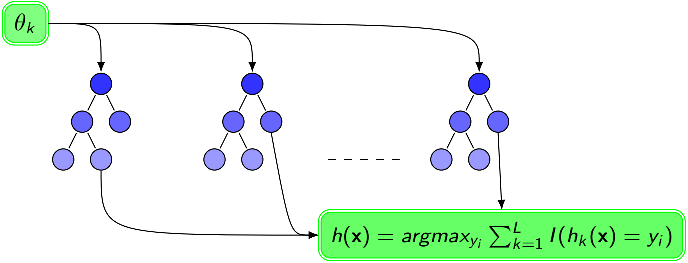
Definition [Breiman 2001][1]

Definition

A random forest is a classifier consisting of a collection of tree-structured classifiers, noted

$$\{\ h_k = h(x, \theta_k), \quad k = 1, ..., L\ \}$$

where the $\{\theta_k\}$ are independent and identically distributed random vectors and each tree casts a unit vote for the most popular class at input x.

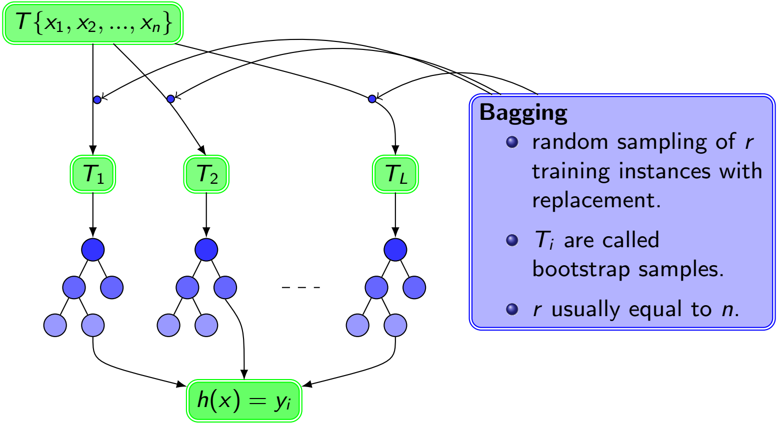


1. L. Breiman, *Random Forests*. Machine Learning, vol.45, num.1, pp 5–32, 2001

Reference algorithm Forest-RI

[Breiman 2001] [1]

Two randomization principles :



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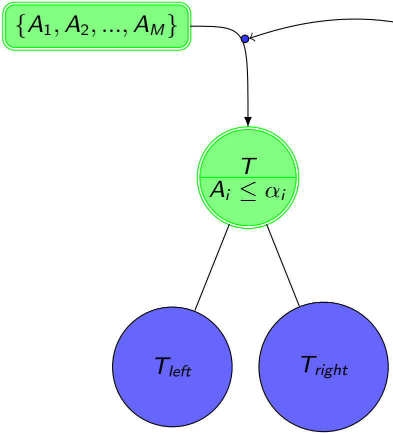


Random Feature Selection (RFS)

- random selection of K features
- find the best split among them
- $K \in [1..M]$

1. L. Breiman, *Random Forests*. Machine Learning, vol.45, num.1, pp 5–32, 2001

How to learn an efficient RF classifier ?

Example : the Madelon dataset (2600 instances, 500 features ($= M$), 2 classes)

Forest-RI : $K = 22$ ($\sqrt{M}$), $L = 300$
$\rightarrow$ test error rate $= 30.50\%$

Forest-RI : $K = 260$, $L = 300$
$\rightarrow$ test error rate $= 17.73\%$

Forest-RI : $K = 260$, $L = 100$ (tree selection)
$\rightarrow$ test error rate $= \mathbf{15.96}\%$

- **Understand how to control these performances**
- **Improve the learning method in consequence**

Random Feature Selection

Control the Randomness

What do we know ?

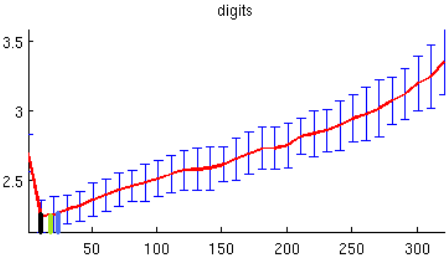
Control of the randomness : hyperparameter K

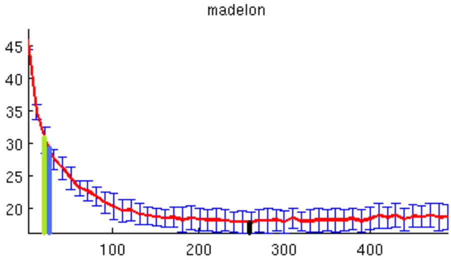
K : number of random features used for each node splitting

- Allow to control the "amount" of randomness used during the induction procedure

K	1	2	...	$M-1$	M
Randomness	*max*		$\leftarrow \oplus \ldots \ominus \rightarrow$		$\emptyset$

- Several arbitrary values in the literature : 1, $\sqrt{M}$, $\lceil log_2(M) \rceil$





Mean test error rate with respect to K
($K = \sqrt{M}$, $K = \lceil log_2(M) \rceil$, $K = K^*$)

Exhaustive search for K^*

Intuition : the best value for K depends on the relevancy of the features [2]

Experimental protocol :

- 20 datasets, 50 random splits Training/Test for each
- McNemar statistical test of significance
- Exhaustive search of K^* : all possible values between 1 and M are tested
  $\rightarrow$ K^* : the best value in average, over the 50 splits
- Measure the *information gain* for each feature (estimate the relevancy)

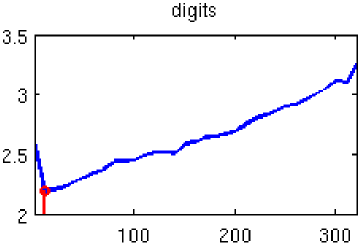
$$Gain(T, A_i) = \Delta I(T, A_i) = I(T) - I(T, A_i)$$

$$I(T) = \sum_{j=1}^{c} -\frac{n_{j.}}{n_{..}} \log_2 \frac{n_{j.}}{n_{..}} \qquad I(T, A_i) = \sum_{k=1}^{m_i} \frac{n_{.k}}{n_{..}} I(T_k))$$

2. Geurts et al, "Extremely Randomized Trees", Machine Learning, 2006

Exhaustive search for K^*

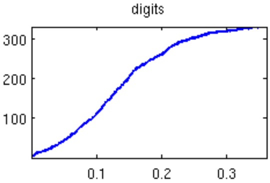
2 types of results :



Mean error rates w.r.t. K

17 of the 20 datasets

- $K = \sqrt{M}$ is always a good choice
- high proportion of relevant features



Cumulative number of features w.r.t. to information gain

Exhaustive search for K^*

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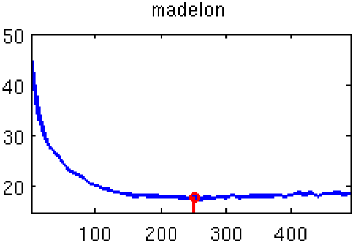
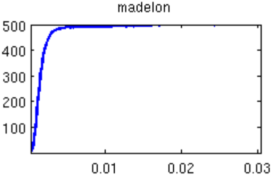


Mean error rates w.r.t. K

3 of the 20 datasets

- None of the given settings is satisfying
- Very few relevant features



Cumulative number of features w.r.t. to information gain

The Forest-RK algorithm

An alternative with a new push-button algorithm, called Forest-RK [3]

Forest-RK

- Same as Forest-RI, *i.e.* combines Bagging and RFS.
- Except that the value of K is randomly set for each node.

  $\rightarrow$ K is not an hyperparameter of the method anymore.
  $\rightarrow$ increase diversity by allowing some trees to be "less random"

3. S. Bernard, L. Heutte, S. Adam, *Forest-RK : A New Random Forest Induction Method*, ICIC, 2008.

The Forest-RK algorithm
Evaluation

Forest-RK $\equiv$ Forest-RI/$K_{\sqrt{M}}$ on the 17 "regular" datasets

For the 3 "atypical" datasets (with very few relevant features) :

	DigReject	Madelon	Musk
Forest-RI/K^*	7.12 ± 0.34	17.73 ± 1.60	2.34 ± 0.30
Forest-RI/$K_{\sqrt{M}}$	7.58 ± 0.34	30.50 ± 1.94	2.40 ± 0.29
Forest-RK	7.16 ± 0.33	18.34 ± 1.52	2.34 ± 0.31
Test de McNemar			
RK vs RI/$K_{\sqrt{M}}$	RK	RK	$\equiv$
RK vs RI/K^*	$\equiv$	$\equiv$	$\equiv$

$\rightarrow$ Forest-RK $>$ Forest-RI/$K_{\sqrt{M}}$

Forest-RK $\equiv$ *Forest-RI/K^** **for all the 20 datasets**

Dynamic Tree Induction

Control the Diversity

What do we know ?

Generalization error convergence

For an increasing number of trees in the forest, generalization error rate converges to a minimum.[1 4 5]

- **Strength** : $s = E_{X,Y}[mr(X,Y)]$

  where $mr(X,Y) = P_\Theta(h(X,\Theta) = Y) - max_{j \neq Y} P_\Theta(h(X,\Theta) = j)$ is the margin of the forest

- **Correlation** : $\overline{\rho} = E_{\Theta,\Theta'}[\rho(rmg(\Theta,X,Y), rmg(\Theta',X,Y))]$

  where $rmg(\Theta,X,Y) = I(h(X,\Theta) = Y) - I(h(X,\Theta) = \hat{\jmath}(X,Y))$ is the raw margin of a tree

- **Generalization error bound** :

$$PE^* \leq \frac{\overline{\rho}(1-s^2)}{s^2} \leq \frac{\overline{\rho}}{s^2} \quad (1)$$

1. L. Breiman, *Random Forests*. Machine Learning, vol.45, num.1, pp 5–32, 2001
4. Latinne et al., *Limiting the Number of Trees in Random Forests*, MCS, 2001.
5. Bernard et al., *Using Random Forests for Handwritten Digit Recognition*, ICDAR, 2007.

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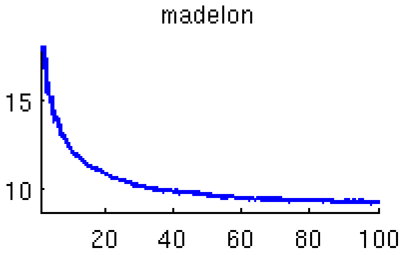

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Analysis of several sub-forests

What makes an ensemble of trees more accurate than another ?

$\rightarrow$ Generate different sub-forests and examine their performances

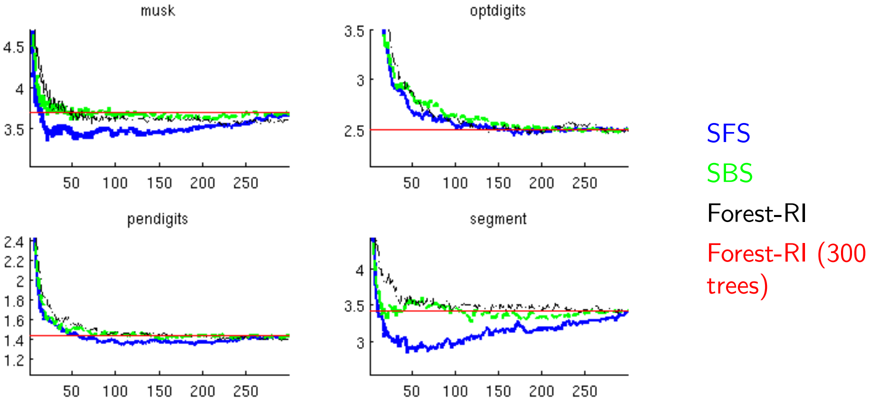
Experimental protocol :

- 20 datasets, 50 random splits Training/Test
- Sequential classifiers selection techniques :
  - Sequential Forward Search (SFS)
  - Sequential Backward Search (SBS)

  $\rightarrow$ Selection criteria : validation error rate

Analysis of several sub-forests

Results



Error rate w.r.t. the number of trees in the sub-forests

→ 18 datasets : at least one sub-forest significantly better than the forest

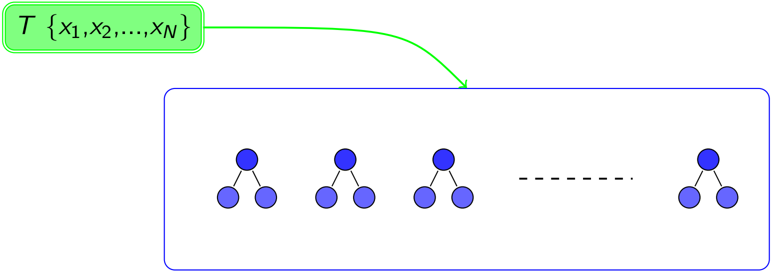
→ Sometimes only 10% of the trees can reach the performance of the forest

Dynamic Random Forest (DRF)[1]

Principe

Key idea : guide the tree induction

→ New tree grown to suit the best possible to the current sub-forest.



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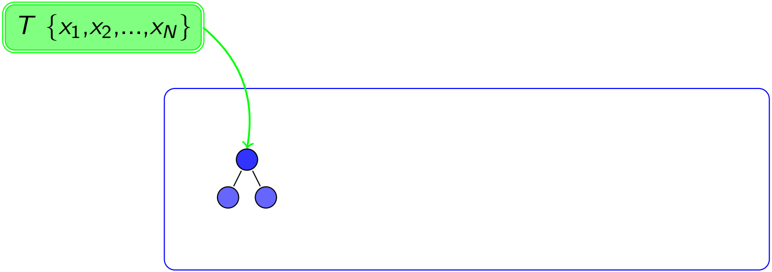

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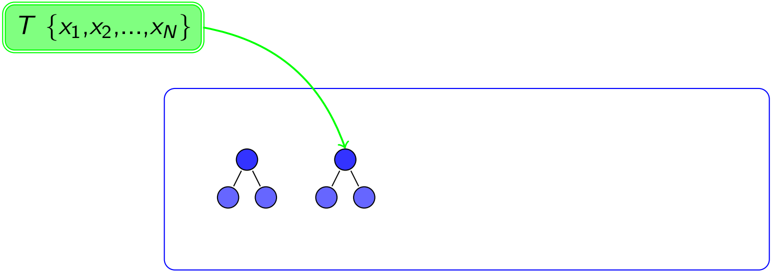

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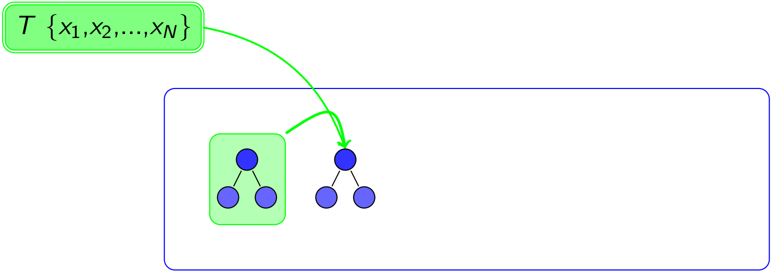

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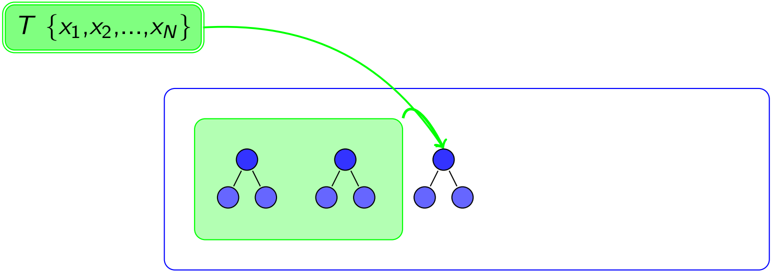

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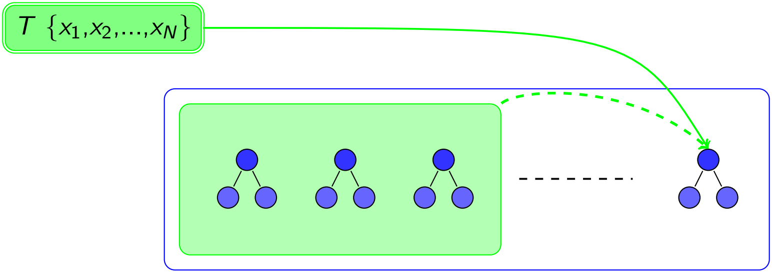

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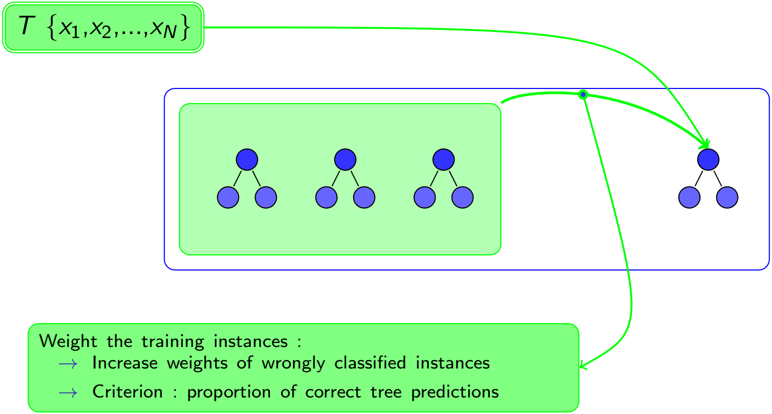

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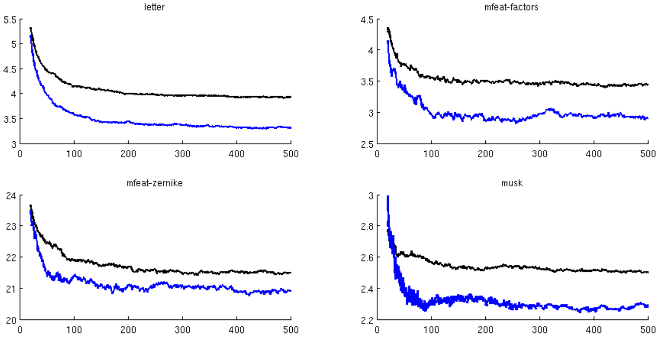
Dynamic Random Forest
Evaluation



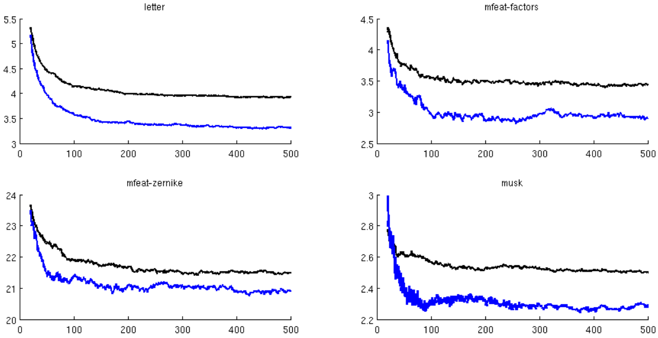
DRF

Forest-RK

Test error rate w.r.t. the number of trees

1. 500-trees Forests :
   → DRF significantly better than Forest-RK for 14 datasets
   → DRF > Forest-RK > *Forest-RI/K**

Dynamic Random Forest
Evaluation



DRF

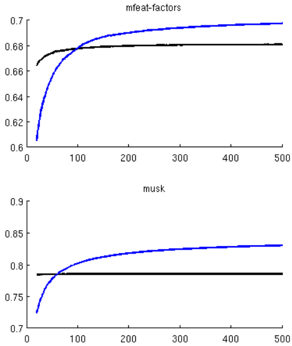
Forest-RK

Test error rate w.r.t. the number of trees

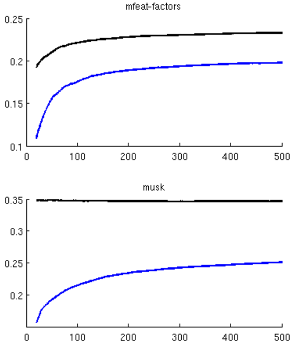
2. Generalization error convergence
   $\rightarrow$ Not mathematically proven anymore, but experimentally observed

Strength and Correlation in Dynamic Random Forest

The *strength* must be maximized and the *correlation* must be minimized (for minimizing $\frac{\overline{\rho}}{s^2}$)



Strength w.r.t. the number of trees



Correlation w.r.t. the number of trees

DRF, Forest-RK

Works in progress with Random Forests

In the chronological order...

1. **Dynamic Random Forest** : weighting the features to guide the Decision Tree induction
2. **One-Class Random Forest** [2] : Random Forests for One-Class classification
3. **Random Forests with Random Hierarchies** : Randomization principle for Hierarchical Multilabel classification
4. **Cost-Sensitive Random Forests** : Random Forests for Cost-Sensitive classification with multi-objective evolutionnary techniques

2. Désir et al. *One-Class Random Forests*, Pattern Recognition, 2013