

# A Survey of Comparison-Based System-Level Diagnosis

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The growing complexity and dependability requirements of hardware, software, and networks demand efficient techniques for discovering disruptive behavior in those systems. Comparison-based diagnosis is a realistic approach to detect faulty units based on the outputs of tasks executed by system units. This survey integrates the vast amount of research efforts that have been produced in this field, from the earliest theoretical models to new promising applications. Key results also include the quantitative evaluation of a relevant reliability metric—the diagnosability—of several popular interconnection network topologies. Relevant diagnosis algorithms are also described. The survey aims at clarifying and uncovering the potential of this technology, which can be applied to improve the dependability of diverse complex computer systems.

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## 1. INTRODUCTION

Consider a large computer system composed of several units. This can be any system: a multicomputer combining several processors on a single chip, a network or a system running on that network, a large software system. It is highly likely that units of those systems will eventually misbehave, producing results not expected from the unit's specification. This may occur for a number of reasons, including typical hardware and software faults as well as malicious interference. If those faults cause the system to fail as a whole, users can be severely affected. Comparison-based diagnosis is a realistic approach to detect disruptive behavior in such systems. This diagnosis paradigm is a solid long-standing theory that has found several diverse applications across the years.

Comparison-based diagnosis relies on comparisons of task outcomes produced by system units. The set of all comparison results is called the system syndrome, or comparison syndrome. The first models were proposed by Malek [1980], and by Chwa and Hakimi [1981b]. These models assume the existence of a central observer which collects information about comparisons and then performs the diagnosis of the system

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based on comparison results, thus determining which units are faulty. Another model, proposed by Maeng and Malek [1981]—the MM model—assumes that comparisons are executed by the units themselves, and only comparison results are sent to the central observer, which then completes the diagnosis of the system. Maeng and Malek [1981] also present a special case of the MM model, called MM\*, in which a node executes comparisons for all its connected neighbors.

Sengupta and Dahbura in [1992] generalize the MM model by allowing comparators to be one of the units being compared. They also give a characterization of diagnosable systems under the MM model. Probabilistic comparison-based models were first introduced by Dahbura et al. [1987], and this method was applied to multiprocessor diagnosis. Probabilistic models assume a fault probability, that is, the probability that a unit produces an incorrect output. Blough and Brown in [1999] presented the broadcast comparison model, a fully distributed comparison-based system-level diagnosis model based on reliable broadcast. In this model a task is assigned to a pair of different nodes which execute the task and broadcast their outputs to all nodes. All fault-free nodes execute all comparisons and diagnose the system. Other comparison-based models that are fully distributed but do not employ a reliable broadcast primitive were presented in Albini and Duarte Jr. [2001] and Ziwich et al. [2005], in which fault-free nodes test and classify the system nodes in sets.

Diagnosis algorithms are employed to determine which units are faulty given the comparison syndrome. Relevant diagnosis algorithms are described in the survey. Sengupta and Dahbura [1992] proposed an  $O(N^5)$  diagnosis algorithm under the MM\* model to identify faults, where  $N$  is the number of system units. Recently, an  $O(N \times \Delta^3 \times \delta)$  diagnosis algorithm, where  $\Delta$  and  $\delta$  are respectively the maximum and the minimum degrees of a node, was proposed by Yang and Tang in [2007].

Besides the diagnosis itself, another important related problem is the diagnosability, that is, determining the largest number  $t$  of faulty units that do not make it impossible to diagnose the system. This is a relevant quantitative measure of the reliability of a given system. Key contributions described in this article include carefully proven results on the diagnosability of several popular interconnection network topologies under comparison-based models. We describe the results for hypercubes and enhanced hypercubes [Wang 1999], butterflies [Araki and Shibata 2002a], crossed cubes [Fan 2002], locally twisted cubes and hypercube-like networks [Yang and Yang 2007; Chiang and Tan 2007], star graphs [Zheng et al. 2002], matching composition networks [Lai et al. 2004], and  $t$ -connected and product networks [Chang et al. 2004]. Recently the strong diagnosability of several topologies was presented by Sheu et al. [2008] and Hsieh and Chen [2008a, 2008b].

This survey aims at integrating and clarifying the vast amount of research efforts that have been produced in this field. Traditional applications [Yang and Tang 2007] include the detection of faults in multicomputers, such as multiprocessor interconnection networks. Potential emerging areas of application include the many-core computers that are due in the near future, which will combine hundreds of microprocessor cores on a single chip and need to be tested and diagnosed efficiently. Comparison-based approaches have already been shown to be well-suited to multicomputer systems [Wang et al. 1994a] and have also been applied to diagnosing defects in chips at the wafer-scale integration level [Rangarajan et al. 1990; Fussell et al. 1989]. These prior successes make the approach a strong candidate for use in many-core systems.

Other new promising applications have appeared recently. The new applications include identifying faults in mobile ad hoc networks [Elhadeif et al. 2007; Chessa and Santi 2001], checking the integrity of distributed replicated data [Ziwich et al. 2005], and checking the manipulation of job results by malicious nodes in grid computing platforms [Martins et al. 2006b]. The objective of the survey is to uncover the potential

of this technology, hopefully leading to novel applications in diverse complex computer systems.

The rest of the article is organized as follows. Section 2 introduces system-level diagnosis. Section 3 presents the first comparison-based diagnosis models. The MM and the MM\* models, their characterization, and algorithms are presented in Section 4. Section 5 presents the diagnosis and diagnosability of hypercubes and enhanced hypercubes. The next six sessions describe results for butterflies, crossed cubes, locally twisted cubes and hypercube-like networks, star graphs, matching composition networks, and  $t$ -connected and product networks. Section 12 presents recent strong diagnosability results for several topologies. Section 13 describes the broadcast comparison model. Section 14 introduces the generalized distributed comparison-based models. Sections 15 and 16 describe probabilistic and evolutionary comparison-based approaches, respectively. Section 17 describes the application of comparison-based diagnosis to mobile ad hoc networks. Section 18 summarizes relevant results and is followed by concluding remarks.

## 2. SYSTEM-LEVEL DIAGNOSIS

As faults are inevitable in every computer system, it is important to be able to determine which units of the system are working and which are faulty. The first model proposed for automatic system-level diagnosis was the PMC model [Preparata et al. 1967], named after the author's initials: Preparata, Metzger, and Chien.

The PMC model assumes a system  $S$  that consists of a set of  $N$  independent units,  $u_0, u_1, \dots, u_{N-1}$ . Alternatively unit  $u_i$  is also referred to as *unit  $i$*  or *node  $i$* , or even *processor  $i$* . Each unit  $u_i$  is assumed to be in one of two states, *faulty* or *fault free*.

Diagnosis is based on the ability of units to test the status of other units [Masson et al. 1996; Jalote 1994]. A unit is tested as a whole, it is not possible to test part of a unit, and the state of a unit does not change during diagnosis. In the PMC model a test involves the controlled application of stimuli and the observation of the corresponding responses from the tested unit. Preparata et al. [1967] define a test as a “diagnostic program” tailored for each system.

The PMC model assumes that a fault-free unit executes tests and reports test results reliably, that is, a fault-free tester can always correctly determine whether the tested unit is faulty or fault free. More precisely, on the basis of the responses to the stimuli, the outcome of the test is classified as *pass* or *fail*, nevertheless the authors highlight that more detailed information about the failure may be retained for further investigation.

While fault-free units are assumed to be able to execute tests correctly, no assumptions are made about tests executed by faulty units, that is, they may produce incorrect test outcomes [Preparata et al. 1967; Hakimi and Amin 1974]. The set of tests is called the *connection assignment*, and the set of all test outcomes is called the *syndrome* of the system. The syndrome is processed by an external entity which diagnoses the system, that is, determines the state of all system units.

The model employs a directed graph in order to represent the connection assignment. The vertices of this graph are the system units, and there is an edge directed from unit  $i$  to unit  $j$  if unit  $i$  tests unit  $j$ . Each edge is labeled by the test outcome,  $a_{i,j} = \{0, 1\}$ . If unit  $i$  tests unit  $j$  as faulty, then  $a_{i,j} = 1$ , on the other hand if unit  $i$  tests unit  $j$  as fault free, then  $a_{i,j} = 0$ . These results hold only if the tester is fault free, otherwise the test outcomes are unreliable.

Depending on the number of faulty units and on the testing assignment, it is impossible to correctly diagnose the system. A system is defined as *one-step  $t$ -diagnosable* if all faulty units within the system can be identified when the number of faulty units is not greater than  $t$ . Furthermore, a system is defined as *sequentially  $t$ -diagnosable* if at least one faulty unit can be identified and be repaired or replaced, so that the testing

can be continued using the repaired unit to eventually diagnose all faulty units in the system.

In  $t$ -diagnosable systems the problem of determining the maximum value for  $t$  is called the diagnosability problem. Preparata et al. gave necessary conditions for  $t$ -diagnosability of their model [Preparata et al. 1967]. Later Hakimi and Amin [1974] characterized the PMC model, and proved that if no two units test each other, each unit is tested by at least  $t$  others, and  $N \geq 2t + 1$ , these are not only necessary but also sufficient conditions for a system to be  $t$ -diagnosable.

Another early model for system-level diagnosis is the BGM model, also named after its authors' initials: Barsi, Grandoni, and Maestrini [1976]. This model is similar to the PMC model, employs the same testing graph, but assumes different test outcomes. Its basic assumptions are: each test is executed by a single unit; each unit has the capability of testing any other unit; no unit tests itself; and, for any pair of units  $u_i, u_j$ , unit  $u_i$  performs at most one test on unit  $u_j$ . The diagnostic model is defined as follows:

- if  $u_i$  is fault free, the test outcome is 0 if  $u_j$  is fault free; the test outcome is 1 if  $u_j$  is faulty;
- if  $u_i$  is faulty and  $u_j$  is fault free, both test outcomes are possible; and
- if  $u_i$  and  $u_j$  are faulty, the test outcome is necessarily 1.

In this model if the test outcome  $a_{i,j} = 0$ , that is, unit  $i$  tests unit  $j$  as fault free, then it is possible to conclude that  $u_j$  is *not* faulty; while if  $a_{i,j} = 1$ , then it is not possible that both  $u_i$  and  $u_j$  are fault free. No other possibilities can be excluded given the results of the test executed by unit  $i$  on unit  $j$ . The BGM model also gives the necessary and sufficient conditions for  $t$ -diagnosability both for one-step diagnosis and sequential diagnosis. If each unit is tested by at least  $t$  other units, and  $N \geq t + 2$ , they show that the one-step diagnosability is at most  $N - 2$ . In sequential diagnosis also, called diagnosis with repair, if a faulty unit is found, it is repaired and the process is then sequentially repeated until all faulty units are diagnosed and repaired; the sequential diagnosability of arbitrary topology networks has been shown to be co-NP-complete [Raghavan and Tripathi 1991]. Later [Albini et al. 2004] the diagnosability of symmetric graphs under the BGM model was also determined.

An important result in system-level diagnosis was the introduction of *adaptive* diagnosis [Nakajima 1981; Hakimi and Nakajima 1984]. Previous models consisted of initially choosing the set of tests to be executed, then executing those tests, and finally evaluating the test results in order to identify all faulty units. In adaptive diagnosis, the set of tests to be executed is dynamically determined, based on the results of previous tests. The first adaptive diagnosis model was introduced by Nakajima [1981]. Assuming a system  $S$  of  $N$  units with no more than  $t$  faulty units, the proposed model adaptively chooses and executes tests, repeating the process until a fault-free unit is identified. Then this unit is employed as a tester from which all faulty units are identified. It is proved that  $(N - 1) + t(t + 1)$  tests are sufficient to identify all faulty units in such a system.

In adaptive diagnosis and all other previous models, test results are collected and processed by an external entity which determines the state of all system units. In *distributed* system-level diagnosis, proposed by Kuhl and Reddy [1980, 1981] and Kuhl [1980], the fault-free nodes of the system themselves diagnose the state of all nodes. These nodes execute tests and exchange test results with each other. They proposed the SELF distributed system-level diagnosis algorithm that, although fully distributed, is nonadaptive, that is, each unit has a fixed testing assignment. Later Hosseini et al. [1984] extended the SELF algorithm, introducing the NEW-SELF algorithm which allows all fault-free nodes to independently diagnose the state of all nodes, provided the total number of failures does not exceed a given bound  $t$ . The EVENT-SELF algorithm

was then proposed by Bianchini et al. [1990], which uses event-driven techniques to reduce the amount of network resources required for diagnosis.

The adaptive distributed system-level diagnosis algorithm, *Adaptive-DSD*, was proposed by Bianchini and Buskens [1991, 1992]. Adaptive-DSD is, at the same time, distributed and adaptive. Adaptive-DSD is executed at each node of the system at predefined *testing intervals*. Each node is tested only once per testing interval. A *testing round* is defined as the period of time in which all nodes of the system have executed their assigned tests at least once. All fault-free nodes achieve consistent diagnosis in at most  $N$  testing rounds. Up to  $N - 1$  nodes may be faulty so that fault-free nodes are still able to diagnose the system.

Each time the algorithm is executed on a fault-free node, it performs tests on other nodes until another fault-free node is found, or the tester runs out of nodes to test. The testing graph is thus a ring connecting fault-free nodes. When the tester executes a successful test, that is, the tested node is fault free, the tester obtains diagnostic information from the tested node. Let the *diagnosis latency* be the number of testing rounds required by all fault-free nodes to complete the diagnosis of the system. Adaptive-DSD has a worst-case latency of  $N$  testing rounds. Adaptive-DSD was implemented and practical results were presented showing the effectiveness of the algorithm when used to monitor a real Ethernet network.

Hierarchical diagnosis was proposed in order to reduce the latency of adaptive distributed diagnosis [Duarte Jr. and Nanya 1995; 1998; Duarte Jr. et al. 2000]. In hierarchical diagnosis, nodes are grouped in progressively larger virtual clusters so when a fault-free node is tested, the tester obtains information about all nodes in that cluster. The Hierarchical Adaptive Distributed System-level Diagnosis (*Hi-ADSD*) algorithm [Duarte Jr. and Nanya 1998] has a diagnosis latency of at most  $\log_2^2 N$  testing rounds for a system of  $N$  nodes. Another hierarchical diagnosis algorithm, *Hi-ADSD* with time-stamps [Duarte Jr. et al. 2000], employs clusters with size  $N/2$  resulting in an average latency shown to be lower than *Hi-ADSD*'s.

Although several models and algorithms for system-level diagnosis assume a subjacent fully connected network, some have been proposed for general topology networks including Bagchi and Hakimi [1991], Stahl et al. [1992], Rangarajan et al. [1995], and Duarte Jr. and Weber [2003]. Subbiah and Blough [2004] define a theoretical framework called *bounded correctness* in which it is possible to prove the correctness of distributed diagnosis in the presence of dynamic faults and repairs. They present both an algorithm for fully connected systems and another for general topology networks and prove their bounded correctness.

### 3. COMPARISON-BASED DIAGNOSIS: THE FIRST MODELS

The first comparison-based diagnosis model was proposed by Malek [1980]. This model assumes that, in a system with  $N$  units, it is possible to compare outputs produced by task executions from some or every pair of units. The unit that performs comparisons is called a comparator. A comparison that results in a mismatch indicates that one or both units are faulty. Note that it is possible that both units being compared are faulty, and in this case the comparison must indicate a mismatch. Thus this model assumes that:

- (1) outputs produced by two fault-free units that execute the same task are always identical;
- (2) the output produced by a faulty unit must be different from the outputs produced by any other unit (faulty or fault free) for the same task.

This model consists of two activities: fault detection and fault location. The objective of fault detection is only to determine the presence of faulty units in the system,

Table I. Possible Comparison Outcomes of Malek's [1980] Model

Unit 1	Unit 2	Comparison Outcome
fault-free	fault-free	0 (pass)
fault-free	faulty	1 (fail)
faulty	fault-free	1 (fail)
faulty	faulty	1 (fail)

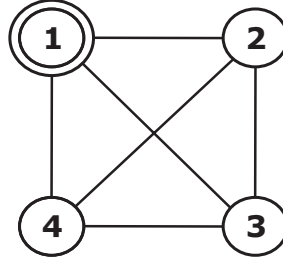


Fig. 1. A graph example representing a system with four units; unit 1 is faulty.

Table II. All Comparison Outcomes for a System with 4 Units When Unit 1 Is Faulty

Unit Id	Unit Id	Comparison Outcome
1	2	1 (fail)
2	3	0 (pass)
3	4	0 (pass)
1	3	1 (fail)
1	4	1 (fail)
2	4	0 (pass)

but it is not possible to determine which units are faulty. Fault location allows the identification of faulty units. The fault model assumed can be best described as the *incorrect computation fault* [Barborak et al. 1993; Laranjeira et al. 1991]. This type of fault occurs when a unit does not produce the correct result in response to the correct inputs.

A system with  $N$  units is modeled as a graph  $G = (V, E)$  that is a connected graph, that is, there is a path between any pair of vertices and there may be one or more edges between any given pair of vertices. In this graph,  $V$  is a set of  $N$  vertices and  $E$  is a set of edges. Each vertex of set  $V$  corresponds to a processor or a system unit. Each edge in  $E$  represents the communication connection or link between a pair of units.

This model assumes that the tasks are executed by pairs of different units. It also assumes that a central observer exists which collects and maintains information about the task outputs. This central observer also performs the diagnosis of the system based on comparison results, determining which are the system's faulty units. The central observer is a trustful reliable unit that never fails. When the outputs of two units are compared, the possible outcomes are shown in Table I. The set of possible comparison outcomes is also called the *invalidation rule*. The outcome *pass* indicates that both units are fault free, while *fail* indicates that at least one of the units is faulty. In this case, more comparisons are necessary to identify the faulty unit.

It is proved that, in a system with  $N$  units in which comparisons of every pair of units is possible, the maximum number of faulty units is  $N - 2$  for the diagnosis to be correct, that is, the diagnosability is  $N - 2$ . As an example, Figure 1 shows a complete graph  $G$  with four vertices and six edges. Considering unit 1 as faulty, Table II shows the comparison outcomes of every possible comparison in this system.

Table III. Possible Comparison Outcomes of Chwa and Hakimi's [1981b] Model

Unit 1	Unit 2	Comparison Outcome
fault-free	fault-free	0 (pass)
fault-free	faulty	1 (fail)
faulty	fault-free	1 (fail)
faulty	faulty	0 or 1

Chwa and Hakimi in [1981b] proposed another comparison-based diagnosis model, similar to Malek's [1980] model. In this model the system consisting of  $N$  units is also represented by graph  $G = (V, E)$ . The same task is also sent to the units in pairs. The units' states—fault free or faulty—are determined by the task output comparisons. If the comparison results in a mismatch, it indicates the presence of at least one faulty unit, as shown in Table III. This model also assumes a central observer which performs the complete diagnosis of the system based on the comparison outcomes.

The difference between this model and the previous one is that when two faulty units receive the same task to execute, they may produce the same outputs, that is, the comparisons of these two tasks outputs may result in a match.

In Barborak et al. [1993] Barborak, Malek, and Dahbura survey the first comparison-based diagnosis models. This is a key paper in which diagnosis is treated in a unifying framework together with other distributed problems and algorithms, including consensus and the Byzantine Generals problem. Among the contributions of Barborak et al. [1993], a detailed fault classification is given, including the specification of the incorrect computation fault model, which best defines the faults that can be handled by comparison-based diagnosis. This is relevant because several early diagnosis papers only implicitly present the assumed fault model, by specifying how faults are detected. The survey also argues that if the frequency in which two units become faulty is low, then there is a low probability that they will be faulty at the same time. Thus two units executing the same tasks should produce identical results unless one, or both, have become faulty.

### 3.1. Early Models: Extensions and Evaluations

Ammann and Dal Cin in [1981] also investigated the diagnosability of comparison-based diagnosis, showing that a necessary condition for a system to be  $t$ -diagnosable is that each node in the testing graph has degree at least  $t$ ; a minimum degree strictly greater than  $t$  is a sufficient condition. The degree—or order—of a node is the number of edges incident on this node. Later Ammann and Dal Cin also presented an algorithm for sequential diagnosis of a subset of  $t$ -diagnosable systems. The complexity of the proposed algorithm is  $O(N^2)$ . They also introduced a parallel algorithm for the diagnosis when the topology is a tree [Dal Cin 1982; Ammann and Dal Cin 1981].

Yang and Masson in [1987] present a comparison model considering multiprocessor fault diagnosis applied for  $t_1/t_1$ -diagnosable systems. The system is said to be  $t/s$ -diagnosable if, in the presence of at most  $t$  faults, all the faulty units can be identified by replacing at most  $s$  units [Friedman 1975]. The  $t_1/t_1$ -diagnosable systems is a special case of  $t/s$ -diagnosable systems when  $s = t_1$  [Chwa and Hakimi 1981a]. In a  $t_1/t_1$ -diagnosable system all faulty units, except one or none, can be correctly identified, that is, at most one fault-free unit can be incorrectly diagnosed as faulty. As in Chwa and Hakimi's [1981a] model, the model by Yang and Masson [1987] assumes that the comparison of two faulty units may result in a match. They also present an  $O(|C|)$  algorithm under the  $t_1/t_1$ -diagnosis comparison model.

Xu and Huang [1990] characterized the  $t/(N - 1)$ -diagnosability of several types of structures under Chwa and Hakimi's [1981a] model. A system with  $N$  units is

$t/(N - 1)$ -diagnosable if at most  $t$  units are faulty and the faulty units are in a set of size  $(N - 1)$ , that is, at least one unit exists such that it can be unambiguously identified as fault free. They present a synthesis of optimal  $t/(N - 1)$ -diagnosable configurations for several topologies, such as chains and loops. In particular it was shown that for  $N = 2t + 1$ , chains are  $t/(N - 1)$ -diagnosable if  $N \leq 9$  and loops are diagnosable for  $N \leq 13$ . Later Xu and Randell [1997] applied  $t/(N - 1)$  diagnosis to the software design process. They proposed  $t/(N - 1)$ -variant programming scheme which diagnoses faults in a redundant software framework.

Kozłowski and Krawczyk [1991] extend Chwa and Hakimi's [1981a] diagnosis model for hybrid fault situations. A hybrid fault situation is defined to be  $t/m$ -restricted if the number of faulty units does not exceed  $t$  and the number of misleading comparison outcomes is less than  $m$ . A misleading comparison outcome occurs when a fault-free unit evaluates a faulty unit as fault free. Kozłowski and Krawczyk [1991] also present an  $O(N|C|)$  algorithm for comparison-based diagnosis under a hybrid fault situation.

Fuhrman and Nussbaumer in [1996a, 1996b] present the Bounded Symmetric Comparison (BSC) model for comparison-based system-level diagnosis. This model is based on Chwa and Hakimi's model [1981b] but includes a limit on the number of nodes that can produce identical faulty results. In the BSC model  $f_1$  represents the maximum number of nodes that can be faulty, and  $f_2$  is the upper bound on the number of faulty nodes that can produce identical faulty results. Furthermore,  $f_2 \leq f_1$ . The authors prove necessary and sufficient conditions for one-step diagnosability of a system under the BSC model. They show that a system is one-step diagnosable if and only if for every two distinct sets  $F_1, F_2$  where  $F_1 \subset V, F_2 \subset V$  and  $|F_1| \leq f_1, |F_2| \leq f_1$  one of the following conditions is satisfied.

- There is an edge between a node in  $V - (F_1 \cup F_2)$  and a node in  $(F_1 \cup F_2) - (F_1 \cap F_2)$ .
- One component of the graph corresponding to either  $F_1 - (F_1 \cap F_2)$  or  $F_2 - (F_1 \cap F_2)$ .

In Sallay et al. [1999] faults affecting the comparator and the central observer are considered. In order to try to diagnose the comparators, the authors propose a strategy to exhaustively run comparisons of fault-free units and comparators. These tests are performed with different input tasks and it is assumed that a faulty unit always produces the same response for the same input task. The authors apply their proposed approach to wafer-scale circuits, presenting a simple cost-effective wafer design solution.

Kreutzer and Hakimi [1983] and Lombardi [1986] present two comparison-based models—called KH1 and KH2—that consider faults of comparator units apart from faults of other tested units. In the first model the comparison of task outcomes produced by two faulty units may match, and in the second model if the task outcomes comparison match, both units are considered fault free. Pelc [1992] argues that these models are in fact equivalent to those of Chwa and Hakimi [1981a, 1981b] and Malek [1980]. Kreutzer and Hakimi [1983] also present the characterization for a system to be  $(t - t_c)$ -diagnosable under these models, where a  $(t - t_c)$ -diagnosable is a system with at most  $t$  faulty units and at most  $t_c$  faulty comparators. They show that a system  $S$  is  $(t - t_c)$ -diagnosable if and only if  $S$  is  $t$ -diagnosable and  $t_c < |\Gamma(i)|/2$ , where  $\Gamma(i) \leq \Gamma(j) \mid \forall j \in V$  and  $\Gamma(i) = \{j \mid i \text{ and } j \text{ are compared}\}$ .

Pelc in [1992] performed an algorithmic analysis of both Malek's [1980] and Chwa and Hakimi's [1981a, 1981b] comparison models, which he calls asymmetric and symmetric models respectively. In the analysis he presents the worst-case number of tests for optimal algorithms for  $t$ -diagnosis, sequential  $t$ -diagnosis, and one-step  $t$ -diagnosis for both models. He also considers nonadaptive and adaptive testing and shows that using adaptive testing the number of tests is often smaller.



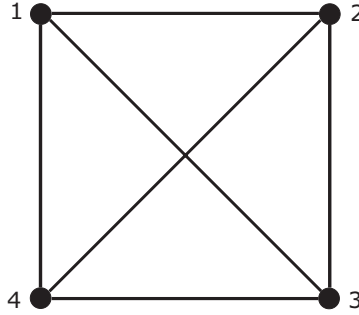


Fig. 2. An example graph representing a system with 4 units.

The minimum number of tests for completing  $t$ -diagnosis,  $t \leq N$ , under Malek's model [1980] is  $\lceil N/2 \rceil$ . In case of sequential  $t$ -diagnosis (identifies at least one faulty unit), where  $t \leq N - 2$ , the minimum number of tests required is  $\text{MAX}(\lfloor N/2 \rfloor * t) + 1$  when an adaptive testing strategy is employed and  $N - \lfloor N/(t + 2) \rfloor$  for nonadaptive diagnosis. In case of adaptive one-step  $t$ -diagnosis (identifies all faulty units in one step), when  $t \leq N - 2$  the minimum number of tests is  $\theta(N^2/(N - t))$  and when  $N \geq 2t + 1$  the number of tests is  $\lfloor N/2 \rfloor + 3.5\lceil t/2 \rceil + 3$ . For nonadaptive one-step  $t$ -diagnosis,  $t \leq N - 2$  the minimum number of tests is  $\theta(Nt)$ .

The minimum number of tests for completing  $t$ -diagnosis, where  $t \leq N - 1$  for Chwa and Hakimi's [1981a, 1981b] model is  $N - \lfloor N/(t + 1) \rfloor$ . In case of sequential  $t$ -diagnosis, where  $t < N/2$  the minimum number of tests required is  $N - \lceil N/(t + 1) \rceil + 1$  when an adaptive testing strategy is employed and  $N - \lfloor N/(2t + 1) \rfloor$  for nonadaptive diagnosis. In case of one-step adaptive  $t$ -diagnosis, when  $t < N/2$  the minimum number of tests is  $\theta(N)$  [Kreutzer and Hakimi 1983]. For nonadaptive one-step  $t$ -diagnosis, if  $t < N/2$  the minimum number of tests is  $\theta(Nt)$ .

#### 4. MM AND MM\* COMPARISON-BASED DIAGNOSIS MODELS

The MM model was proposed by Maeng and Malek [1981] for systems composed of multiprocessor systems consisting of homogeneous processors. The system is represented as a graph  $G = (V, E)$ , where  $V$  is the set of units and  $E$  is the set of communication links. Figure 2 shows an example graph for a system with four units. In the MM model, the states of the units are determined by comparing the task output of one unit with the output generated by another unit for the same task. The main difference of the MM model to the previous models [Malek 1980; Chwa and Hakimi 1981b] is that it allows the comparisons to be performed by the units themselves, that is, units are also comparators. A unit  $k$  is a comparator of units  $i$  and  $j$  only if  $(k, i) \in E$  and  $(k, j) \in E$ ; furthermore  $k \neq i$  and  $k \neq j$ . Comparison results are still sent to a central observer that achieves the complete diagnosis.

A diagnosable system under the MM model is represented by a multigraph  $M = (V, C)$  defined over the same set of units of graph  $G$ . Each edge  $(i, j)_k \in C$  represents the outputs from units  $i$  and  $j$  compared by another unit  $k$ .  $M$  is a multigraph because the outputs from each pair of units may be compared by more than one unit of the system, that is, more than one edge may exist between the same pair of vertices. Figure 3 shows a multigraph  $M$  defined over the graph in Figure 2. As an example, in this multigraph the comparison  $(3, 4)_1$  is performed, that is, the outputs of units 3 and 4 are being compared by unit 1. Furthermore, the two edges between units 1 and 2 show that these units are compared by two other units: unit 3 and unit 4.

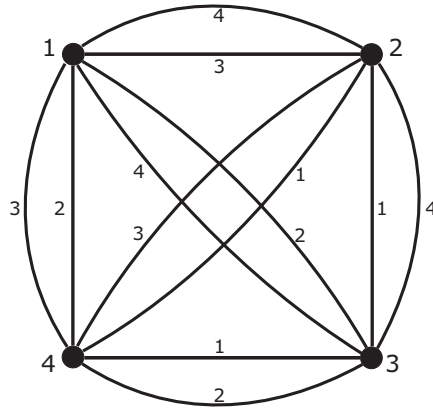
Fig. 3. A multigraph  $M$  for a system with 4 units.

Table IV. Possible Comparison Outcomes for the MM Model

Comparator	Unit 1	Unit 2	Comparison Outcome
fault-free	fault-free	fault-free	0 (pass)
fault-free	fault-free	faulty	1 (fail)
fault-free	faulty	fault-free	1 (fail)
fault-free	faulty	faulty	1 (fail)
faulty	fault-free	fault-free	0 or 1
faulty	fault-free	faulty	0 or 1
faulty	faulty	fault-free	0 or 1
faulty	faulty	faulty	0 or 1

The notation  $r((i, j)_k)$  is used to represent the comparison result of units  $i$  and  $j$  by unit  $k$ . The result is 0 when the comparison matches and the result is 1 when the comparison indicates a mismatch. If  $r((i, j)_k) = 1$ , at least one of the units  $i$ ,  $j$  or  $k$  is faulty. If the result is 0 and the tester  $k$  is fault free, then  $i$  and  $j$  are also fault free. But if the tester  $k$  is faulty, the comparison outcomes are not reliable and it is not possible to obtain any conclusion about the state of units  $i$  and  $j$ .

The main assumptions of the MM model are as follows.

- Every fault is permanent, that is, units cannot recover from faults.
- A comparison performed by any faulty unit is unreliable.
- Two faulty units executing the same task always generate different outputs.
- Each faulty unit generates incorrect outputs for every input task, that is, the comparison of the task outputs by a faulty unit and any other unit (faulty or fault free) always results in a mismatch.
- An upper bound  $t$  exists, that is the maximum number of units in the system that may be faulty so the diagnosis can be achieved.

Consider  $(i, j)_k$ , if unit  $k$  is fault free, a mismatch of the comparison indicates that at least one unit is faulty and  $r((i, j)_k) = 1$ . If the comparison matches, both units are fault free and  $r((i, j)_k) = 0$ . If unit  $k$  is faulty the comparison outcome is unreliable. The set of all comparison results is also called the syndrome of the system, which is represented by  $\sigma$ . All possible comparison outcomes are shown in Table IV.

Figure 4 shows an example set of comparison outcomes for multigraph  $M$  of Figure 3. Each edge has two labels one represents the unit that compares the task outputs from the units connected by each edge, the other represents the result of each comparison

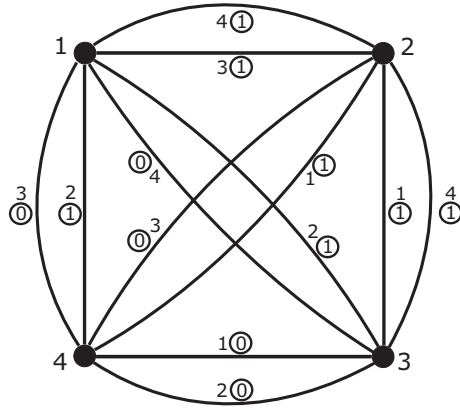


Fig. 4. Multigraph  $M$  showing comparison outcomes for the example system.

and it is shown within a circle. As an example, units 3 and 4 are being compared by unit 1 and the comparison output indicates a match.

Besides presenting the comparison-based diagnosis model, it is shown in Maeng and Malek [1981] that in order to correctly diagnose a system with one faulty node ( $t = 1$ ) the total number of units in the system ( $N$ ) must be greater than 3. For  $t \geq 2$ ,  $N$  must be greater than or equal to  $2t + 1$ . The necessary and sufficient conditions for one-step  $t$ -diagnosability under this model are: (1) the degree of every node must be at least  $t$ ; (2) for every pair of nodes  $i, j$  whose distance is 1 or 2 at least  $t$  nodes of a set called  $W_{i,j}$  must be removed in order to disconnect any such pair of nodes and their neighbors from the rest of the graph, furthermore (3) there is no  $W_{i,j}^*$  if  $W_{i,j}$  has exactly  $t$  nodes.  $W_{i,j}^*$  is defined as a  $W_{i,j}$  that has at least a pair of vertices  $r, s$  for which  $W_{i,r} = (W_{i,j} - r) \cup j$  and  $W_{j,s} = (W_{i,j} - s) \cup i$ .

Maeng and Malek in [1981] also give the procedure that follows to construct the minimal graph for diagnosing system  $S_{t,N}$ , with  $t \geq 4$  and  $N = 2t + 1$ . For  $t = 1$  or 2, the minimal graph is the complete graph. For  $t = 3$  the authors show that number of edges must be at least 14.

- (1) If  $t$  is even, then let  $t = 2r$ .  $S_{2r,N}$  has edges connecting vertices  $i, j$  such that  $i - r \leq j \leq i + r$ , modulo  $N$ . Nodes have sequential identifiers starting from zero.
- (2) If  $t$  is odd and  $N$  is even, then let  $t = 2r + 1$ .  $S_{2r+1,N}$  has an edge connecting vertex  $i$  to vertex  $i + (N/2)$  in  $S_{2r,N}$ ,  $1 \leq i \leq N/2$ .
- (3) If  $t$  is odd and  $N$  is odd, then let  $t = 2r + 1$ .  $S_{2r+1,N}$  has all edges in  $S_{2r,N}$  plus edges from vertex 0 to vertex  $(N - 1)/2$  and to vertex  $(N + 1)/2$  and from vertex  $i$  to vertex  $i + (N + 1)/2$  for  $1 \leq i < (N - 1)/2$ .

Figure 5 shows  $S_{4,9}$ .

A discussion on the diagnosis latency under the MM model is also given. First it is assumed that each comparator can execute only one comparison per time unit. A *test cycle* is defined as one application of the maximum number of comparisons to a system, which consist of  $\lfloor N/3 \rfloor$  simultaneous comparisons, as each unit is either a comparator or is compared. It is shown that the lower bound for the minimum number of test cycles is  $\lceil \lceil Nt/2 \rceil / \lfloor N/3 \rfloor \rceil$ , where  $\lceil Nt/2 \rceil$  is the minimal number of comparisons when each vertex has degree  $t$ .

The MM\* model is a special case of the MM model [Maeng and Malek 1981; Sengupta and Dahbura 1992], the only difference is that each unit compares every pair of neighbor units with which they are connected. For example, Figure 6 shows a system where

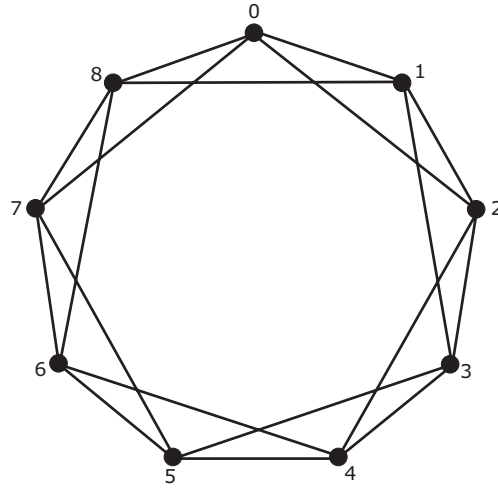
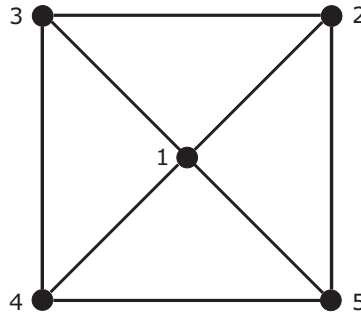
Fig. 5. The minimal graph  $S_{4,9}$ .

Fig. 6. An example system with 5 units.

the tester, unit 2, under the  $MM^*$  model, performs all the following comparisons  $(1, 3)_2$ ,  $(1, 5)_2$ , and  $(3, 5)_2$ . The comparison outcomes are then sent to the central observer that will complete the diagnosis.

Sengupta and Dahbura in [1992] generalize the MM model by allowing comparators to be one of the units being compared. They also give a characterization of diagnosable systems under the MM model. It is worth pointing out that when the comparator always compares itself with another unit, the comparison assignment is equivalent to the test assignment of the PMC model when the tester performs a test on the other unit. In this sense this model generalizes the PMC model.

Furthermore, they present a polynomial-time algorithm to identify the faulty processors in a general system in which each processor carries out comparisons for every pair of neighbors. Most important, they show that the diagnosability of general systems under this model is NP-complete.

#### 4.1. $t$ -Diagnosability

Sengupta and Dahbura [1992] solve the problem of whether a given system is  $t$ -diagnosable for some integer  $t$  under the MM model. Let  $S_1$  and  $S_2$  be sets of units. A pair  $(S_1, S_2)$  such that  $S_1, S_2 \subset V$  and  $|S_1|, |S_2| \leq t$  is defined as either *distinguishable* or *indistinguishable* as follows. Let  $\sigma(F)$  be the set of syndromes which could be

produced if  $F$  is the set of faulty nodes. The pair of sets  $S_1, S_2 \mid S_1 \neq S_2$  is said to be indistinguishable if and only if  $\sigma(S_1) \cap \sigma(S_2) \neq \emptyset$ ; otherwise it is distinguishable.

To prove that a pair  $(S_1, S_2)$  is distinguishable, at least one of the following three conditions must be satisfied.

- (1)  $\exists i, k \in V - S_1 - S_2$  and  $\exists j \in (S_1 - S_2) \cup (S_2 - S_1)$  such that  $(i, j)_k \in C$
- (2)  $\exists i, k \in S_1 - S_2$  and  $\exists k \in (V - S_1 - S_2)$  such that  $(i, j)_k \in C$
- (3)  $\exists i, k \in S_2 - S_1$  and  $\exists k \in (V - S_2 - S_1)$  such that  $(i, j)_k \in C$

Sengupta and Dahbura [1992] prove that a system  $S$  with  $N$  nodes is  $t$ -diagnosable if and only if for each pair of sets  $S_1, S_2 \in V \mid S_1 \neq S_2$  and  $|S_1|, |S_2| \leq t$ ,  $(S_1, S_2)$  is a distinguishable pair. In other words, considering set  $\sigma(S_1)$  which is the set of syndromes that can be produced if  $S_1$  is the set of faulty nodes and considering the analogously defined set  $\sigma(S_2)$ ,  $\sigma(S_1) \cap \sigma(S_2) = \emptyset$ .

They also prove that for a system with  $N$  nodes to be  $t$ -diagnosable,  $N \geq 2t + 1$  and each node has degree at least  $t$ , that is, the output of each node must be compared to at least  $t$  outputs from other nodes. Furthermore, for each set  $X \subset V$  such that  $|X| = N - 2t + p$  and  $0 \leq p \leq t - 1$ , they prove that  $|T(X)| \geq p$ , where  $T(X) = \{j \mid (i, j)_k \in C \text{ and } i, k \in X\} - X$ . In other words: the number of nodes in subset  $V - X$  that is compared to some node in  $X$  and by some node in  $X$  is at most  $t - 1$ .

#### 4.2. A Polynomial-Time Comparison-Based Diagnosis Algorithm

Sengupta and Dahbura [1992] propose a polynomial-time algorithm, with complexity  $O(N^5)$ , for comparison-based diagnosis under the MM\* model. Given a system represented by graph  $G = (V, E)$  whenever  $(i, j), (i, k) \in E$  node  $i$  compares the results of nodes  $j$  and  $k$ .

The algorithm adaptively determines the comparisons to be executed on the basis of comparison results. A node  $i$  running this algorithm starts comparing two nodes  $j, k \mid (i, j), (i, k) \in E$ , that is, node  $i$  performs the comparison  $(j, k)_i$ . If the comparison outcome  $r((j, k)_i) = 1$  (mismatch), then node  $i$  chooses another different pair of nodes to compare, if there is such a pair. If the comparison outcome  $r((j, k)_i) = 0$  (match), then node  $i$  uses node  $j$  in order to compare *all* its neighbors, that is, all comparisons  $(j, p)_i \mid (p, i) \in E$ .

The diagnosis algorithm by Sengupta and Dahbura [1992]—called DIAGNOSIS—is shown in Figure 7. The algorithm receives as input the set of all comparison outcomes, the system's syndrome  $(\sigma)$ . Some definitions are required to understand the algorithm and are given next.

Given graph  $G' = (V', E')$ ,  $K \subseteq V'$  is a vertex cover set of  $G'$  if every edge in  $E'$  is incident to at least one vertex in  $K$ . This concept can be extended to hypergraphs [Berge 1973], employing hyperedges instead of edges. A vertex cover set of minimum cardinality is called a minimum vertex cover set.

A subset  $M \subseteq E'$  is called a matching if no vertex in  $V'$  is incident to more than one edge in  $M$ , without any self-loop. A matching of maximum cardinality is called a maximum matching.

A set  $X \subset V$  is called an Allowable Fault Set (AFS) of system  $S$  for syndrome  $\sigma$ , if for any three nodes  $i, j, k$  such that  $(i, j)_k \in C$ :

- if  $k \in V - X$  and  $i, j \in V - X$  then  $r((i, j)_k) = 0$ ;
- if  $k \in V - X$  and  $\{i, j\} \cap X \neq \emptyset$  then  $r((i, j)_k) = 1$ .

For syndrome  $\sigma$ , an AFS of minimum cardinality is called a Minimum AFS of  $\sigma$ —MAFS( $\sigma$ ), and  $N(i) = \{j \mid (i, j) \in E\}$  is the set of neighbors nodes of  $i$ .

In the initialization phase, the set of Faulty nodes ( $F$ ) is set to empty, and  $S(\sigma)$  is computed.  $S(\sigma)$  is the set of comparators that returned mismatches for all executed

```

Algorithm DIAGNOSIS

/* Initialization Phase */
 $F \leftarrow \emptyset$ ;
compute  $S(\sigma)$ ;

for each node  $i \in S(\sigma)$  such that  $|N(i)| = t + 1$  do
  /* First Step */
  for each  $k \in N(i)$  do
    if  $N(i) - \{k\}$  is an AFS (Allowable Fault Set) then
       $F \leftarrow N(i) - k$ ;
      stop the algorithm;
    end if
  end for
end for

for each node  $i \in S(\sigma)$  such that  $|N(i)| = t$  do
  /* Second Step */
  for each  $k \in N(i)$  do
    if  $N(i)$  is an AFS then
       $F \leftarrow N(i)$ ;
      stop the algorithm;
    end if
  end for

  /* Third Step */
  compute  $H(\sigma)$ ;
  for each  $k \in N(i)$  do
    for each  $h \in H(\sigma)$  do
      if  $N(i) - k + h$  is a vertex cover of hypergraph  $Z = (V, H(\sigma))$  then
         $F \leftarrow N(i) - k + h$ ;
        stop the algorithm;
      end if
    end for
  end for
end for

/* Fourth (and Final) Step */
begin
  construct graph  $Y = (V, M(\sigma))$ ;
  remove all self-loops in  $Y$ ;
  compute the maximum matching of  $Y$ ;
   $F \leftarrow$  the minimum vertex cover set  $Y$ ;
end

```

Fig. 7. The DIAGNOSIS algorithm by Sengupta and Dahbura [1992].

comparisons. Please remember from the diagnosability results that every node in system has degree at least  $t$ .

$S(\sigma)$  cannot have a node  $i$  such that  $|N(i)| > t + 1$ : these nodes are fault free because the number of faulty nodes is at most  $t$ . If there are two fault-free nodes  $j$  and  $k$  in  $N(i)$ , then  $r((j, k)_i) = 0$ .

Now every node  $i$  in  $S(\sigma)$  such that  $|N(i)| = t + 1$  is examined. If removing a node  $k$  from  $N(i)$  results in an AFS, then the set of faulty nodes  $F = N(i) - k$ . If this is the case, the algorithm then stops.

In case  $F$  has not been determined in the previous step, then every node  $i$  in  $S(\sigma)$  such that  $|N(i)| = t$  is examined. First the algorithm checks whether  $N(i)$  is an AFS: in this case  $F = N(i)$ , and the algorithm stops. Otherwise, there are nodes outside  $N(i)$  that can be faulty. In order to check these nodes, hypergraph  $Z = (V, H(\sigma))$  is created, where set  $H(\sigma)$  is as follows. Initially  $H(\sigma) = \{(i, j, k) \mid (i, j)_k \in C \text{ and } r((j, k)_i) = 1\}$ . Then the following step is executed until  $H(\sigma)$  does not change: if  $\{i, j, k\} \in H(\sigma)$  and  $m$  has tested  $k$  as fault free and  $\{i, j, m\} \notin H(\sigma)$  then  $\{i, j, m\}$  is added to  $H(\sigma)$ .

In the next step of the algorithm, each node  $h \in H(\sigma)$  replaces one by one each node  $k$  in  $N(i)$ . The algorithm checks whether the resulting set is a vertex cover of hypergraph  $Z = (V, H(\sigma))$ . In this case, the set of faulty nodes is found,  $F = N(i) - k + h$ . The algorithm then stops.

Finally, if  $F$  has not been found in the previous steps, a new graph  $Y = (V, M(\sigma))$  is constructed, with  $M(\sigma)$  constructed in five steps.

- Step 1.* For any  $i \notin S(\sigma)$ , if  $r((j, k)_i) = 1$  and  $i$  has tested both  $j$  and  $k$  as fault free, then  $(i, i) \in M(\sigma)$ .
- Step 2.* For any  $i \notin S(\sigma)$ , if  $r((j, k)_i) = 1$  and  $i$  has tested  $j$  as faulty, then  $(i, j) \in M(\sigma)$ , and if  $r((j, k)_i) = 1$  and  $i$  has tested  $k$  as faulty, then  $(i, k) \in M(\sigma)$ .
- Step 3.* For any  $i \in S(\sigma)$ , if there exists  $j \in N(i)$  such that  $j \in S(\sigma)$  then  $(i, j) \in M(\sigma)$ .
- Step 4.* For any  $i \in S(\sigma)$ , if there exists  $j \in N(i)$  such that  $j \notin S(\sigma)$  then, if  $i$  has tested  $j$  as fault free, then  $(j, p) \in M(\sigma)$  for all  $p \in N(i) - \{j\}$ , whereas if  $i$  has tested  $j$  as faulty, then  $(i, j) \in M(\sigma)$ .
- Step 5.* For any  $(p, q) \in M(\sigma)$ , if  $p$  has tested  $\alpha$  as fault free and  $q$  has tested  $\beta$  as fault free the  $(\alpha, \beta) \in M(\sigma)$  and  $(p, \beta) \in M(\sigma)$ .

All self-loops are removed and an algorithm for computing the maximum matching for general graphs, such as Micali and Vazirani [1980], is executed on  $Y$ . In the final step the minimum vertex cover set  $F$  of  $Y$  is found using the labeling technique in Dahbura and Masson [1984].

*4.2.1.  $t/x$ -Diagnosability and  $t[x]$ -Diagnosability.* Sengupta and Rhee in [1990] define the  $t/x$ -diagnosability and the  $t[x]$ -diagnosability. A system is  $t/x$ -diagnosable if all faulty processors can be uniquely identified from the set of comparison results whenever there are no more than  $t$  faulty processors and no more than  $x$  missing comparison results. The authors consider the  $t/x$ -diagnosability for cases where the comparison result can be missing possibly because of faulty transmission of the input tasks or of the outputs. A system is  $t[x]$ -diagnosable if all the faulty processors can be uniquely identified from the set of comparison results whenever no more than  $t$  processors are faulty and no more than  $x$  comparison results refer to incorrect identification. This concept is used to represent, for example, nodes with intermittent failures.

Let two sets of processors  $S_1, S_2 \cup V$ ,  $X(S_1, S_2) = \{(i, j)_k \mid k \in S_1 \text{ and } \{i, j\} \subset S_1 \cup S_2 \text{ and } \{i, j\} \cap S_2 \neq \emptyset\}$ . In other words,  $X(S_1, S_2)$  denotes the set of comparisons where the comparator is in  $S_1$  and one of compared processors is in  $S_2$  and the other compared processor is in  $S_1 \cup S_2$ . It is proved that a system is  $t/x$ -diagnosable if and only if, for every  $S_1, S_2 \subset V$ , such that  $|S_1|, |S_2| \leq t$ ,  $CT(V - S_1 - S_2, S_1 - S_2) + CT(V - S_1 - S_2, S_2 - S_1) > x$  where  $CT(S_1, S_2)$  denotes the cardinality of the set  $X(S_1, S_2)$ . It is also proved that a system is  $t[x]$ -diagnosable if and only if: (a) for every  $S_1 \subset V$ , such that  $|S_1| = t$ , and for every  $i \in S_1$ ,  $CT(V - S_1, \{i\}) > x$ ; and, (b) for every  $S_1, S_2$ , such that  $S_1, S_2 \subset V$ , and  $|S_1| = |S_2| = t$ , at least one of the following conditions is satisfied.

- $CT(V - S_1 - S_2, S_1 - S_2) > x$
- $CT(V - S_1 - S_2, S_2 - S_1) > x$

**4.2.2. Other Extensions to the MM Model.** In Chen et al. [1993] an extension to the MM model is presented. This model considers both processor and comparator faults separately. Thus, a processor either executes tasks or performs comparisons. It is shown that the system diagnosability is  $t \leq \lfloor \delta/2 \rfloor$ , where  $\delta$  is the minimum degree of nodes in the system. However, they also show that if the number of faulty comparators is less than the number of faulty processors, the diagnosability reaches  $t \leq \delta$ . The authors also present an optimal  $O(|E^*|)$  algorithm for the diagnosability  $t \leq \lfloor \delta/2 \rfloor$ , and an  $(|E^*|^2)$  algorithm for the diagnosability  $t \leq \delta$ , where  $E^*$  is the set of comparators.

Wang et al. in [1994a, 1994b] present new necessary and sufficient conditions for a system to be  $t$ -diagnosable under a comparison-based model based on both the MM model and Sengupta and Dahbura's [1992] model. They show that a system is  $t$ -diagnosable if and only if for all  $Z \subseteq V$  with  $Z \neq \emptyset$ , and for all  $Z_1, Z_2$  that partition  $Z$ ,  $|N_1(Z)| + |N_2(Z)| + CMVC(G_3(Z)) + \max(|Z_1| + |Z_2|) > t$ , where:  $N_1(Z) = \{v \in V - Z \mid \exists z \in Z \text{ with } (v, z)_v \in C\}$ , that is, processors in  $V - Z$  that compare themselves with at least one processor in  $Z$ ;  $N_2(Z) = \{u \in V - Z - N_1(Z) \mid \exists v, w \in Z_1 \text{ with } (v, w)_u \in C\}$ , that is, processors in  $V - Z - N_1(Z)$  that compare two processors in  $Z_1$ ;  $G_3(Z) = (N_3(Z), E_3(Z))$  such that  $N_3(Z) = \{u \in V - Z - N_1(Z) - N_2(Z) \mid \exists v \in Z \text{ and } w \in V - Z - N_1(Z) - N_2(Z) \text{ with } (v, w)_u \in C \text{ or } (u, v)_w \in C\}$ ; and,  $CMVC(G_3(Z))$  denotes the cardinality of a minimum vertex cover set of  $G_3(Z)$ . The authors also present an algorithm for this model and conduct experimental simulations where it is shown that within reduced number of tests the algorithm diagnosis the system provided the number of faults is relatively small.

In Maestrini and Santi [1995] the authors present a correct but incomplete diagnosis algorithm based on the MM model. This algorithm can be applied to locate faults in bidimensional processor arrays, where processors are interconnected in horizontal and vertical meshes.

### 4.3. An $O(N \times \Delta^3 \times \delta)$ Comparison-Based Diagnosis Algorithm

Yang and Tang in [2007] present a diagnosis algorithm for the MM\* model with time complexity  $O(N \times \Delta^3 \times \delta)$ , where  $\Delta$  and  $\delta$  are respectively the maximum and the minimum degrees of a node. This algorithm is introduced as an alternative to Sengupta and Dahbura's [1992]  $O(N^5)$  algorithm. The authors argue that realistic diagnosable systems, such as massive multicomputers, are sparsely interconnected. When  $\Delta, \delta \ll N$  Yang and Tang's [2007] algorithm will behave better than Sengupta and Dahbura's [1992].

The algorithm involves not only the comparison-based diagnosis model, but also the classical PMC model. Initially the *comparison syndrome*, that is, that syndrome that contains comparison outcomes, is evaluated in order to try to find the set of faulty nodes. If diagnosis does not complete using the comparison syndrome, then it is converted to a PMC *test syndrome*, on which a classical diagnosis algorithm is applied in order to obtain the set of faulty nodes. The following definitions are required to understand the algorithm.

Let  $\sigma$  be the comparison syndrome of the system.  $N(i)$  is the neighborhood of node  $i$  and  $|N(i)| = d(i)$  is the degree of  $i$ . For two adjacent nodes  $u$  and  $v$ ,  $v$  is an  $\sigma$ -0 son of  $u$  if there exists  $w \in N(u)$  such that  $r((v, w)_u) = 0$ , that is,  $v$  is a  $\sigma$ -0 son of  $u$  if node  $u$  evaluates node  $v$  as fault free. A node is an  $\sigma$ -0 comparator if it has at least one  $\sigma$ -0 son; otherwise,  $v$  is a  $\sigma$ -1 son of  $u$ . In other words, a node  $u$  is an  $\sigma$ -1 comparator if  $r((v, w)_u) = 1$  for every nodes  $v, w$  that are compared by node  $u$ .  $COMP_1$  denotes the set of all  $\sigma$ -1 comparators.  $COMP_{10}$  denotes the set of all  $\sigma$ -1 comparators of degree  $t$ .  $COMP_{11}$  denotes the set of all  $\sigma$ -1 comparators of degree  $t + 1$ .  $COMP_{12}$  denotes the set of all  $\sigma$ -1 comparators of degree  $\geq t + 2$ .  $COMP_1 = COMP_{10} \cup COMP_{11} \cup COMP_{12}$ .  $SON_0(u)$  denotes the set of all  $\sigma$ -0 sons of node  $u$ . Node  $u$  is  $\sigma$ -conflicting if  $u$  has two  $\sigma$ -0 sons  $v$  and  $w$  such that  $r((v, w)_u) = 1$ .  $CONF$  denotes the set of all  $\sigma$ -conflicting nodes.



Node  $u$  is an  $\sigma$ -0 parent of  $v$  if there exists a node  $w$  such that  $r((v, w)_u) = 0$ .  $\text{PARENT}_0(v)$  denotes the set of all  $\sigma$ -0 parents of node  $v$ , and  $\text{PARENT}_0(U) = \bigcup_{x \in U} \text{PARENT}_0(x)$ . Node  $u$  is an  $\sigma$ -0 predecessor of  $v$  if there exists a sequence of nodes  $w_0 = u, w_1, \dots, w_p, w_{p+1} = v$  such that  $w_i \in \text{PARENT}_0(w_{i+1})$ , for  $i = 0, 1, \dots, p$ .  $\text{PRED}_0(u)$  denotes the set of all  $\sigma$ -0 predecessors of node  $u$ .  $\text{PRED}_0(U) = \bigcup_{x \in U} \text{PRED}_0(x)$  and  $\text{PRED}_0[U] = \text{PRED}_0(U) \cup U$ .

A set  $U \subset V$  is also called an Allowable Fault Set (AFS) of system  $S$ , if for any three nodes  $u, v, w$  where  $(v, w)_u \in C$ ,  $u \in V - U$ ,  $v, w \in N(u)$ , and  $v \neq w$ , such that:

- if  $v, w \in U - X$  then  $r((v, w)_u) = 0$ ;
- if  $\{v, w\} \cap X \neq \emptyset$  then  $r((v, w)_u) = 1$ .

A  $t$ -AFS of  $\sigma$  is an AFS of  $\sigma$  with at most  $t$  nodes. Let  $K$  be a set of nodes such that  $K \subseteq V$ , a  $K^{+1}$  AFS of  $\sigma$  is an AFS of  $\sigma$  that is of the form  $K \cup u$  for some  $u \in V - K$ .  $\text{NODE}^{+1}(K)$  denotes the set of all  $K^{+1}$  nodes of  $\sigma$ . A  $K^{+1}$  group of  $\sigma$  is a set of three nodes  $u, v, w \in V - K$  such that either  $r((v, w)_u) = 1$  or  $r((w, u)_v) = 1$  or  $r((u, v)_w) = 1$ .  $\text{GROUP}^{+1}(K)$  denotes the set of all  $K^{+1}$  groups of  $\sigma$ .

Let  $\sigma$  be the comparison syndrome of a system  $G$  and  $H$  a subsystem of  $G$ , the restriction of  $\sigma$  on  $H$ , denoted as  $\sigma|_H$ , is a comparison syndrome on  $H$  defined by  $(v, w)_u$  for all  $u, v, w \in V(H)$ ,  $v, w \in N(u)$ , and  $v \neq w$ , where  $V(H)$  is the set of vertices of graph  $H$ . The  $\sigma$ -induced test syndrome, denoted by  $t[\sigma]$ , is a test syndrome on  $G$  defined in this way: for any two adjacent nodes  $u$  and  $v$ , let  $t[\sigma](u, v) = 0$  or 1 according to whether  $u$  is an  $\sigma$ -0 parent of  $v$  or not.

The diagnosis algorithm by Yang and Tang [2007]—called  $\text{MM}^*$  DIAG—is shown in Figure 8. Figure 9 shows the procedure CHECK\_IF that is used by the algorithm. The algorithm receives as input the graph  $G(V, E)$  that represents the  $t$ -diagnosable system and the system's comparison syndrome. The algorithm produces as output the node's faulty set.

The algorithm is divided in three phases. In the first phase the algorithm defines the sets  $\text{COMP}_{10}$ ,  $\text{COMP}_{11}$  and identifies the set  $\text{COMP}_{12}$ , that is, defines all the  $\sigma$ -1 comparators. The authors prove, by contradiction, that all comparators in set  $\text{COMP}_{12}$  are faulty. For every comparator  $x$  in the sets  $\text{COMP}_{11}$  or  $\text{COMP}_{10}$ , the algorithm checks all possible candidates for the unique  $t$ -AFS provided that  $x$  is fault free.

In this context the procedure CHECK\_IF is quite important. This procedure receives as input a system  $G(V, E)$ , the corresponding comparison syndrome  $\sigma$ , and a set  $K \subset V$  that is not an AFS. The procedure returns another set  $K^{+1} = K \cup \{u\} \mid u \in V - K$ , if such set exists. This new set must be an AFS and must have  $|K| + 1$  nodes. Otherwise the procedure returns “No”. If a  $t$ -AFS is found in one of these steps, the diagnosis is complete and the AFS set found is returned as the set of faulty nodes. Otherwise, all  $\text{COMP}_1$  comparators are considered faulty and the algorithm goes to the second phase.

In phase 2, the CONF set is identified, that is, all nodes that have two  $\sigma$ -0 sons  $v$  and  $w$ , but the comparison  $r((v, w)_u) = 1$ . The authors prove, also by contradiction, that all nodes in CONF are faulty. Then, the algorithm defines a new set  $\text{PRED}_0$  based on sets  $\text{COMP}_1$  and CONF. Set  $U = \text{PRED}_0[\text{COMP}_1 \cup \text{CONF}]$  represents all predecessors that tested directly or indirectly some node in the sets  $\text{COMP}_1$  and CONF. The authors also prove that all nodes in  $\text{PRED}_0[\text{COMP}_1 \cup \text{CONF}]$  are faulty.

In the third and last phase, the diagnosis task is converted into that of the PMC model, instead of the  $\text{MM}^*$  model. A subset  $H = G - U$  composed of all nodes that have not yet been identified as faulty in the previous two phases is constructed. In a key step of the algorithm, an induced test syndrome  $t[\sigma|_H]$  based on  $H$  nodes is constructed from the original comparison syndrome  $\sigma$ . The authors then prove that  $H$  is  $(t - |U|)$ -diagnosable under the PMC model and that  $F - U$  is the unique  $(t - |U|)$ -AFS possible given the test syndrome  $t[\sigma|_H]$ . Then, the location of the remaining faulty

```

Algorithm: MM*_DIAG
/* Input:  An MM*  $t$ -diagnosable system  $G(V, E)$ 
           with  $t$ -fault set and a comparison syndrome  $\sigma$  */
/* Output: The faulty set */

begin
  /* First Phase */
  for every node  $u$  of  $G$  do determine  $PARENT_0(u)$  and  $SON_0(u)$ ;
  calculate  $COMP_1$ ,  $COMP_{10}$  and  $COMP_{11}$ ;
  if there exist  $u \in COMP_{11}$  and  $v \in N(u)$  such that
     $N(u) - \{v\}$  is an AFS of  $\sigma$  then
    return  $(N(u) - \{v\})$ ;
  end if
  if there exist  $u \in COMP_{10}$  such that  $N(u)$  is an AFS of  $\sigma$  then
    return  $(N(u))$ ;
  end if
  if there exist  $u \in COMP_{10}$  and  $v \in N(u)$  such that
     $N(u) - \{v\}$  is an AFS of  $\sigma$  then
    return  $(N(u) - \{v\})$ ;
  end if
  if there exist  $u \in COMP_{10}$  and  $v \in N(u)$  such that
    CHECK_IF( $G, N(u) - \{v\}, \sigma$ )  $\neq$  "No." )
    return (CHECK_IF( $G, N(u) - \{v\}, \sigma$ ));
  end if
  /* Second Phase */
  determine CONF;
   $U \leftarrow PRED_0[COMP_1 \cup CONF]$ ;
  /* Third Phase */
  build the subgraph  $H = G - U$ , build the test syndrome  $t[\sigma|_H]$ ;
  find out the minimum AFS  $U'$  of  $t[\sigma|_H]$  by calling Sullivan's algorithm
  return  $(U \cup U')$ ;
end

```

Fig. 8. The MM\*\_DIAG diagnosis algorithm.

nodes is equivalent to finding the minimum AFS of the induced test syndrome. Thus it is possible to find that minimum AFS set applying the  $O(\delta^3 + |E|)$  algorithm presented by Sullivan [1988].

## 5. COMPARISON-BASED DIAGNOSIS FOR HYPERCUBES

The *hypercube* is a well-known and scalable topology for connecting the nodes of a system [LaForge et al. 2003]. Many properties of hypercubes allow high performance and fault tolerance to be easily incorporated into the system. The diagnosability of hypercubes and enhanced hypercubes [Tzeng and Wei 1991] under the comparison-based MM\* diagnosis model was presented by Wang [1999]. The topology of the system is represented by graph  $G = (V, E)$ , where each node  $i \in V$  represents the system nodes and each edge  $(i, j) \in E$  represents a communication link between nodes  $i$  and  $j$ . The comparisons executed in the system are modeled with a multigraph  $M = (V, C)$ .  $V$  represents the set of system nodes, and a labeled edge  $(i, j)_k \in C$ , where  $k$  is the label, connects  $i$  and  $j$  and means that node  $i$  and node  $j$  are under comparison by node  $k$ .

An  $n$ -dimensional hypercube—also called an  $n$ -hypercube or  $H_n$ —can be viewed as a graph  $G = (V, E)$  where  $V$  consists of  $2^n$  nodes, labeled from  $00 \dots 0$  to  $11 \dots 1$  ( $n$  bits). An edge  $(i, j) \in E$  if and only if  $i$  and  $j$  have only one different bit. Following this fact, all nodes have a connection with exactly  $n$  other nodes. If two nodes  $i$  and  $j$  from a  $n$ -hypercube have  $d$  different bits, it is said that these two nodes have *Hamming distance*

```

Procedure: CHECK_IF
/* Input:  A system  $G(V, E)$ ,
           A comparison syndrome  $\sigma$ , and
           A set  $K \subset V$  that is not an AFS of  $\sigma$  */
/* Output: An  $K^{+1}$  AFS of  $\sigma$  if there is, or "No" if not */

begin
  calculate  $\text{NODE}^{+1}(K)$ ;
  calculate  $\text{GROUP}^{+1}(K)$ ;
  if  $|\text{NODE}^{+1}(K)| \geq 2$  then
    return ("No");
  end if
  if  $|\text{NODE}^{+1}(K)| = 1$  and
     if  $K \cup \text{NODE}^{+1}(K)$  is an AFS of  $\sigma$  then
       return  $(K \cup \text{NODE}^{+1}(K))$ ;
     else
       return ("No");
     end if
  end if
  if  $|\text{NODE}^{+1}(K)| = 0$  then
    if  $\cup_{c \in \text{GROUP}^{+1}(K)} c = \Phi$  then
      return ("No");
    else
      if there is  $u \in \cup_{c \in \text{GROUP}^{+1}(K)} c$  such that  $K \cup \{u\}$ 
         is an AFS of  $\sigma$  then
           return  $(K \cup \{u\})$ ;
         else return ("No");
         end if
    end if
  end if
end

```

Fig. 9. Procedure CHECK\_IF employed by algorithm MM\*\_DIAG.

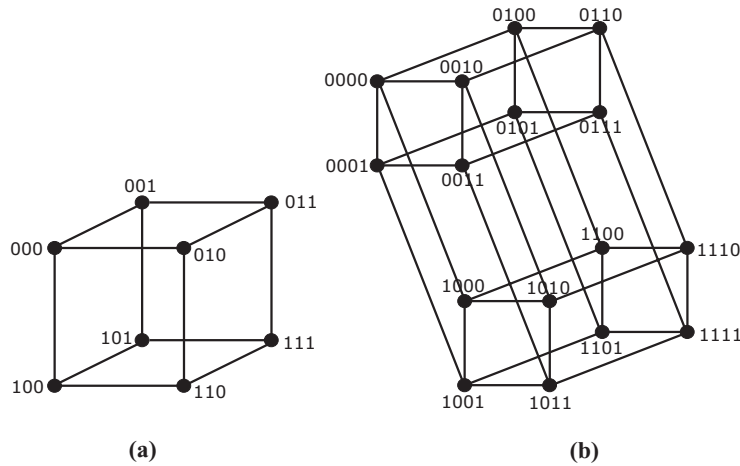


Fig. 10. (a) A 3-hypercube, or simply a 3-cube; (b) a 4-hypercube, or simply a 4-cube.

( $H$ ) equal to  $d$ , denoted as  $H(i, j) = d$ . Then, in a  $n$ -hypercube, or simply  $n$ -cube, there is a connection between  $i$  and  $j$  if and only if  $H(i, j) = 1$ . As an example, Figure 10(a) shows a 3-hypercube and Figure 10(b) shows a 4-hypercube.

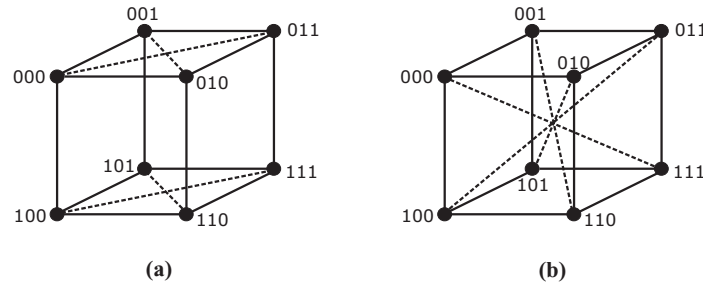


Fig. 11. (a) A (3, 2)-cube; (b) a (3, 3)-cube. The detours are in dotted lines.

In Tzeng and Wei [1991] enhanced hypercubes are defined by adding extra connections links—also known as *skips*—to regular hypercubes. These structures improve the internode distance and the diameter of the system, among other characteristics. An enhanced hypercube is denoted by  $(n, k)$ -cube and is constructed by adding  $2^{n-1}$  extra links to the corresponding  $n$ -cube. There is a skip between a pair of nodes with label  $b_n b_{n-1} \dots b_{k+1} b_k b_{k-1} \dots b_1$  and  $b_n b_{n-1} \dots b_{k+1} \bar{b}_k \bar{b}_{k-1} \dots \bar{b}_1$  where  $b_i$  and  $k \in \{2, \dots, n\}$  is the Hamming distance between the pairs of nodes connected by the skip. Examples of a (3, 2)-cube and a (3, 3)-cube are shown in Figures 11(a) and 11(b), respectively. In this figure the dotted lines correspond to the skips of the enhanced hypercubes.

The diagnosability of  $n$ -hypercubes is proved to be  $n$  under the MM\* model, if  $n \geq 5$  and considering a system with  $N = 2^n$  nodes. The diagnosability of enhanced hypercubes is increased to  $n + 1$  under the same model, if  $n \geq 6$ .

Wang [1999] first defines the vertex cover, which is a subset  $K \subseteq V$  such that every edge of  $E$  is adjacent to one node in  $K$ . The order of vertex  $i$  is then defined as the cardinality of the minimum cover of subgraph  $G_i$ , built with the subset of the nodes that are compared with  $i$  and the corresponding comparison edges.

The proof is based on the characterization previously proposed by Sengupta and Dahbura [1992] that gives a set of conditions that guarantees a system to be  $t$ -diagnosable

- (1)  $N \geq 2t + 1$ , and
- (2) each node has order at least  $t$ , and
- (3) for each  $V' \subset V$ , such  $|V'| = N - 2t + p$  and  $0 \leq p \leq t - 1$ , the number of nodes that are not in  $V'$  but are compared to some node of  $V'$  by some node of  $V'$  is greater than  $p$ .

Condition 1,  $2^n \geq 2n + 1$  is trivially true when  $n \geq 3$ . This condition is valid for both hypercubes and enhanced hypercubes. Condition 2 is satisfied by the proof that every node of an  $n$ -cube has order  $n$  in a hypercube, and order  $n + 1$  in an enhanced hypercube in both cases the order of a node is greater than  $t$ . Finally Wang [1999] shows that the 5-cube is the least hypercube and the  $(6, k)$ -cube are the least enhanced hypercubes satisfying the third of the conditions stated by Sengupta and Dahbura [1992].

In both cases, after the diagnosability is known for hypercubes and enhanced hypercubes, it is possible to apply the  $O(N^5)$  diagnosis algorithm proposed in Sengupta and Dahbura [1992] or the  $O(N \times \Delta^3 \times \delta)$  algorithm proposed in Yang and Tang [2007] to find the faulty nodes of the system. Later in [2003] Yang presents a comparison-based diagnosis algorithm tailored for  $n$ -dimensional hypercubes with  $n \geq 9$  that has time complexity  $O(N \log_2^2 N)$  in the worst case.

## 6. COMPARISON-BASED DIAGNOSIS FOR BUTTERFLIES

The *butterfly* [Rettberg 1986; Leighton 1992] is another topology for interconnection networks which is advantageous for fault-tolerant computing [Leighton et al. 1998;

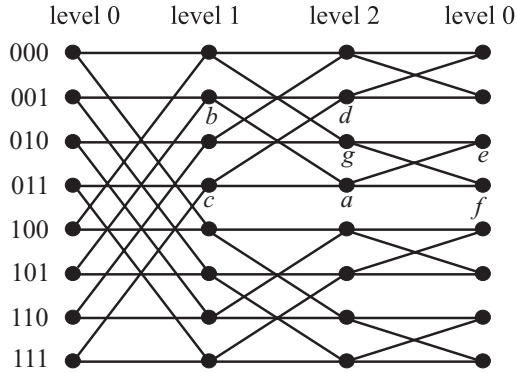


Fig. 12. A  $BF(2, 3)$  butterfly where nodes at level 0 are replicated.

Tamaki 1998]. The diagnosability of butterfly networks under the comparison-based approach is presented by Araki and Shibata [2002a]. This work is also based on Maeng and Malek's [1981] comparison-based diagnosis model, and its motivation is also the fact shown by Sengupta and Dahbura [1992] that it is algorithmically infeasible to compute the diagnosability of general topology networks.

A butterfly network, denoted as  $BF(k, r)$ —also called wrapped butterfly [Leighton 1992]—is a  $k$ -ary  $r$ -dimensional butterfly and has  $rk^r$  nodes. Each node has a label  $\langle \ell; x_0 x_1 \dots x_{r-1} \rangle$ , where  $0 \leq \ell \leq r - 1$ ,  $0 \leq x_i \leq k - 1$ , and  $0 \leq i \leq r - 1$ . The symbol  $\ell$  in the label represents the level of the nodes. The level  $\ell$  is an abstract notation to indicate the column of the node in the butterfly topology representation.

Each node  $\langle \ell; x_0 x_1 \dots x_{r-1} \rangle$  is adjacent to

$$\begin{aligned} &\langle \ell + 1; x_0 \dots x_{\ell-1} y_{\ell} x_{\ell+1} \dots x_{r-1} \rangle \text{ for } 0 \leq y_{\ell} \leq k - 1, \text{ and} \\ &\langle \ell - 1; x_0 \dots x_{\ell-2} y_{\ell-1} x_{\ell} \dots x_{r-1} \rangle \text{ for } 0 \leq y_{\ell-1} \leq k - 1. \end{aligned}$$

As examples, the structure of a  $BF(2, 3)$  and the structure of a  $BF(3, 3)$  are shown in Figure 12 and Figure 13, respectively. In these figures the nodes at level 0 are replicated in the 4<sup>th</sup> column to allow an easier visualization.

In Araki and Shibata [2002a] the authors propose three schemes for the comparison assignment in butterfly networks, the first is called *one-way comparison*, then a *two-way comparison* scheme is defined which is finally improved in the third scheme: *Enhanced Two-Way Comparison (ETWC)*.

In the one-way comparison scheme, each node  $u$  at level  $\ell$  compares every neighbor at level  $\ell + 1$  in a pairwise way. As an example, in the Figure 12, node  $b$  compares  $(a, d)_b$  and the node  $c$  compares  $(a, d)_c$ . As another example, in Figure 13 node  $b$  compares:  $(a, e)_b$ ,  $(a, f)_b$ , and  $(e, f)_b$ . Each node executes  $k(k - 1)/2$  comparisons employing the one-way comparison scheme.

In the two-way comparison scheme, each node  $u$  at level  $\ell$  compares every pair of neighbors at level  $\ell - 1$  and also compares every pair of neighbors at level  $\ell + 1$ . In Figure 12, it is possible to notice that node  $a$  executes two comparisons:  $(b, c)_a$  and  $(c, f)_a$ . In Figure 13 node  $a$  executes six comparisons:  $(b, c)_a$ ,  $(b, d)_a$ ,  $(c, d)_a$ ,  $(x, y)_a$ ,  $(x, z)_a$ , and  $(y, z)_a$ . Each node executes  $k(k - 1)$  comparisons in this scheme.

Araki and Shibata [2002a] show that the diagnosability of a butterfly network  $BF(k, r)$  in which the one-way comparison scheme is employed is  $k - 2$  for  $k \geq 3$  and  $r \geq 3$ . Consider the example in Figure 13. In this  $BF(3, 3)$  butterfly there is a total of 81 nodes, but diagnosis can only be assured if only 1 node is faulty. Araki and Shibata [2002a] show that the two-way comparison scheme improves the diagnosability

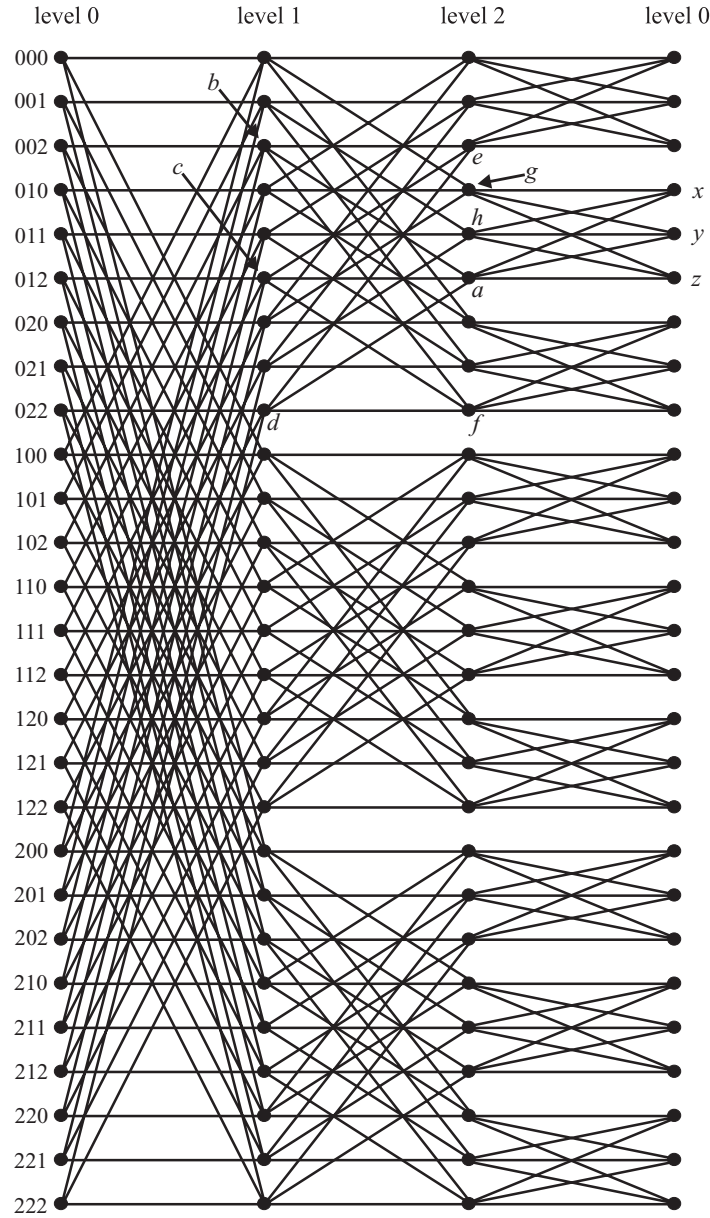
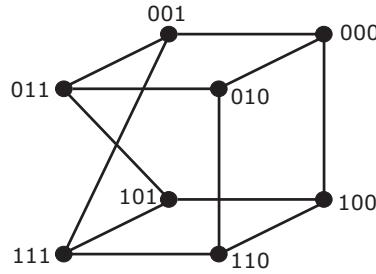


Fig. 13. The structure of a  $BF(3, 3)$  where nodes at level 0 are replicated.

of these networks, which is  $2(k - 2)$ . For the same example, the diagnosability is 2. They then show that the diagnosability of butterflies is at most  $2k$  and propose another comparison scheme that reaches this limit for  $k \geq 2$  and  $r \geq 5$ , the enhanced two-way comparison scheme. In the example, up to 6 nodes may be faulty. Consider that for a node  $u = (\ell; x)$ , ( $x$  is a  $k$ -ary  $r$ -bit string),  $N^+(U) = \{x_0, x_1, \dots, x_{k-1}\}$  is the set of  $k$  nodes adjacent to  $u$  at level  $\ell + 1$  and  $N^-(U) = \{y_0, y_1, \dots, y_{k-1}\}$  is the set of  $k$  nodes adjacent to  $u$  at level  $\ell - 1$ .

Fig. 14. A crossed cube three-dimensional  $CQ_3$ .

A node  $u$  running ETWC performs the following comparisons:

- (1) compares every pair of nodes in  $N^+(U)$ ,
- (2) compares every pair of nodes in  $N^-(U)$ , and
- (3) compares  $x_i$  and  $y_i$  for each  $0 \leq i \leq k - 1$ .

Under the ETWC scheme each node carries out  $k^2$  comparisons. For example, in Figure 13 node  $a$  executes the following nine comparisons:  $(b, c)_a$ ,  $(b, d)_a$ ,  $(c, d)_a$ ,  $(x, y)_a$ ,  $(x, z)_a$ ,  $(y, z)_a$ ,  $(b, x)_a$ ,  $(c, y)_a$ , and  $(d, z)_a$ .

In another paper Araki and Shibata [2002b] propose an  $O(k^2n)$ -time diagnosis algorithm for locating all faulty nodes in a  $BF(k, r)$ . This is better than the  $O(N^5)$  algorithm for general graphs proposed by Sengupta and Dahbura [1992] and the  $O(N \times \Delta^3 \times \delta)$  algorithm proposed in Yang and Tang [2007].

## 7. COMPARISON-BASED DIAGNOSIS FOR CROSSED CUBES

The model proposed by Fan [2002] evaluates the diagnosability of *crossed cubes* under the comparison-based diagnosis model. Crossed cubes are an important variation of hypercubes [Efe 1991, 1992; Efe et al. 1995]. Both crossed cubes and hypercubes are regular graphs that have the same number of nodes, number of edges, and connectivity; and both are recursive in nature. But the diameter of a crossed cube is approximately half the diameter of the corresponding hypercube [Efe 1991; Chang et al. 2000]. The  $n$ -dimensional crossed cube contains a complete binary tree with  $2^n - 1$  nodes and all cycles of length from 4 to  $2^n (n \geq 2)$ ; on the other hand the  $n$ -dimensional hypercube does not have these two properties [Kulasinghe and Bettayeb 1995; Chang et al. 2000].

A node  $x$  in the  $n$ -dimensional crossed cube is a binary string of length  $n$  and is written as  $x_{n-1}x_{n-2} \dots x_0$ . The  $n$ -dimensional crossed cube, also called  $CQ_n$ , is a  $n$ -regular graph with  $N = 2^n$  nodes and  $n2^{n-1}$  edges. Two binary strings  $x = x_1x_0$  and  $y = y_1y_0$  are pair-related, denoted as  $x \sim y$ , if and only if  $(x, y) \in \{(00, 00), (10, 10), (01, 11), (11, 01)\}$ ; if  $x$  and  $y$  are not pair-related, it is denoted  $x \not\sim y$ .

A  $CQ_n$  is defined recursively as presented in Efe [1991, 1992].  $CQ_1$  is the complete graph with two nodes labeled with 0 and 1, respectively. For  $n > 1$ ,  $CQ_n$  consists of two subcubes  $CQ_{n-1}^0$  and  $CQ_{n-1}^1$ . The node  $u = 0u_{n-2} \dots u_0$  of  $CQ_{n-1}^0$  and the node  $v = 1v_{n-2} \dots v_0$  of  $CQ_{n-1}^1$  are adjacent, that is, there is a connection between them, if and only if:

- (1)  $u_{n-2} = v_{n-2}$  if  $n$  is even, and
- (2)  $u_{2i+1}u_{2i} \sim v_{2i+1}v_{2i}$ , for  $0 \leq i < \lfloor \frac{n-1}{2} \rfloor$ .

As an example, Figure 14 shows a three-dimensional crossed cube  $CQ_3$ .

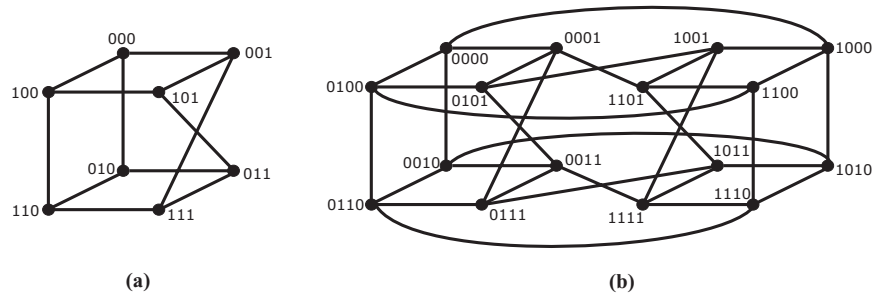


Fig. 15. (a) A  $LTQ_3$  cube; (b) a  $LTQ_4$  cube.

Fan [2002] shows that a crossed cube with  $n \geq 4$  satisfies the conditions stated by Sengupta and Dahbura [1992] for a system to be  $t$ -diagnosable.

- (1)  $2^n \geq 2n + 1$ , and
- (2) each node has degree at least  $n$ , and
- (3) if  $n \geq 4$ , then for each  $V' \subset V(CQ_n)$ , such that  $|V'| = 2^n - 2n + p$  for  $0 \leq p \leq n - 1$  the number of nodes that are not in  $V'$  but are compared to some node of  $V'$  by some node of  $V'$  is greater than  $p$ .

Fan [2002] also proves that the crossed cube with  $n = 4$  is the least that satisfies those conditions, showing that  $CQ_3$  does not satisfy the third condition, while  $CQ_1$  and  $CQ_2$  do not satisfy the second condition. His work concludes that the diagnosability of  $n$ -dimensional crossed cubes is the same as that of  $n$ -dimensional hypercubes, that is, for all  $n \geq 5$  crossed cubes are  $n$ -diagnosable. Nevertheless, for  $n = 4$ , it is also shown that the diagnosability  $CQ_4$  is 4, while a four-dimensional hypercube is not 4-diagnosable.

Either the polynomial algorithm presented in Sengupta and Dahbura [1992] or the algorithm presented in Yang and Tang [2007] can be used to diagnose  $n$ -dimensional crossed cubes if the number of faulty nodes is not greater than  $n$ . Furthermore, Yang et al. in [2005b] present a linear-time comparison-based diagnosis algorithm tailored for crossed cubes with  $n \geq 11$  that has time complexity  $O(N \log_2^2 N)$ .

## 8. COMPARISON-BASED DIAGNOSIS FOR LOCALLY TWISTED CUBES AND HYPERCUBE-LIKE MULTIPROCESSOR SYSTEMS

Yang and Yang [2007] applied comparison-based diagnosis for multiprocessor systems based on *locally twisted cubes*. An  $n$ -dimensional locally twisted cube  $LTQ_n$  [Yang et al. 2005a] is a hypercube variant that has the same number of nodes and edges as an  $n$ -dimensional cube, but has lower diameter and better graph embedding capabilities when compared to a hypercube of the same size [Yang et al. 2005a; 2004; Ma and Xu 2006].

A  $LTQ_n$  is defined recursively as follows [Yang and Yang 2007; Yang et al. 2005a]:

- (1)  $LTQ_2$  is a graph consisting of four nodes labeled with: 00, 01, 10, and 11; respectively connected by four edges: (00, 01), (01, 11), (11, 10), and (10, 00).
- (2) For  $n \geq 3$ ,  $LTQ_n$  is constructed from two disjoint copies of  $LTQ_{n-1}$  according to the following steps:
  - (a) Let  $0LTQ_{n-1}$  denote the graph obtained from one copy of  $LTQ_{n-1}$  by prefixing the label of each node with 0;
  - (b) Let  $1LTQ_{n-1}$  denote the graph obtained from one copy of  $LTQ_{n-1}$  by prefixing the label of each node with 1;



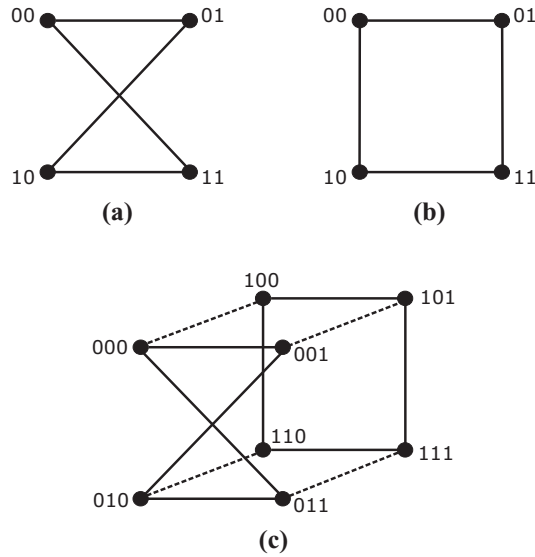


Fig. 16. (a) and (b) Examples of  $HL_2$ ; (c) an example of  $HL_3$ .

- (c) Connect each node  $0x_2x_3 \dots x_n$  of  $0LTQ_{n-1}$  to the node  $1(x_2 \oplus x_n)x_3 \dots x_n$  of  $1LTQ_{n-1}$  with an edge, where  $\oplus$  represents the *xor* binary operation.

As an example, Figure 15(a) shows a three-dimensional locally twisted cube  $LTQ_3$  and (b) shows a four-dimensional locally twisted cube  $LTQ_4$ .

Yang and Yang [2007] present a comparison-based diagnosis algorithm for systems based on the locally twisted cube under the  $MM^*$  model. The algorithm may run in  $O(N \log_2^2 N)$  time if appropriate data structures are employed [Yang and Yang 2007].

Chiang and Tan in [2007] applied the comparison-based diagnosis for *hypercube-like* multiprocessor systems. This class of hypercube-like interconnection networks, also called Hypercube-Like (HL) graphs, was first introduced by Vaidya et al. [1993]. Hypercube-like graphs include the classical hypercubes and many well-known hypercube variants, such as the the twisted cube [Esfahanian et al. 1991], and the multi-twisted cube [Efe 1991].

An  $n$ -dimensional hypercube-like network,  $HL_n$ , can be defined recursively as follows.  $HL_0$  is the graph with one node labeled as 0. For  $n \geq 1$ ,  $HL_n$  consists of two  $HL_{n-1}$  represented by graphs  $G_0$  and  $G_1$ , that is,  $HL_n = \{G_0 \cup G_1 \mid G_0, G_1 \text{ are } HL_{n-1}\}$ .  $HL_n$  has node set  $V(G_0 \cup G_1) = V(G_0) \cup V(G_1)$  and edge set  $E(G_0 \cup G_1) = E(G_0) \cup E(G_1) \cup E_M$ , where  $E_M$  is an arbitrary and perfect matching between the node set of  $G_0$  and  $G_1$  in a one-to-one fashion.

Figure 16(c) shows an example of an  $HL_3$  composed by the two  $HL_2$  in Figure 16(a) and 16(b).

Chiang and Tan [2007] prove that the diagnosability of an  $n$ -dimensional hypercube-like network  $HL_n$  is  $n$  for  $n \geq 5$ .

To prove whether a system is  $t$ -diagnosable, the authors introduce a new concept called local or node diagnosability that is defined as follows. A system  $G(V, E)$  is  $t$ -diagnosable at node  $x \in V(G)$  if, for each pair of distinct sets  $F_1, F_2 \in V(G)$  such that  $|F_1|, |F_2| \leq t$ ,  $F_1 \neq F_2$ , and  $x \in (F_1 - F_2) \cup (F_2 - F_1)$ , the pair  $(F_1, F_2)$  is distinguishable. This is proved using the characterization given by Sengupta and Dahbura [1992].

For every two distinct subsets of nodes  $F_1$  and  $F_2$ ,  $(F_1, F_2)$  is a distinguishable pair, if at least one of the following three conditions must be satisfied.

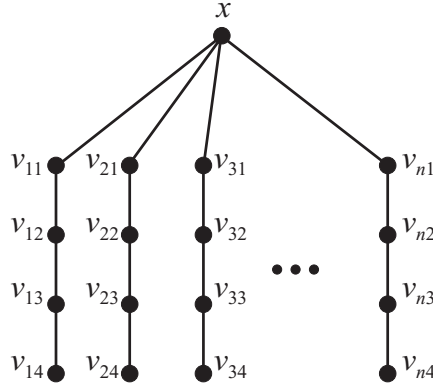


Fig. 17. An extended star structure  $ES(x; n)$  at node  $x$ .

- (1)  $\exists i, k \in V - F_1 - F_2$  and  $\exists j \in (F_1 - F_2) \cup (F_2 - F_1)$  such that  $(i, j)_k \in C$ ;
- (2)  $\exists i, k \in F_1 - F_2$  and  $\exists k \in (V - F_1 - F_2)$  such that  $(i, j)_k \in C$ ;
- (3)  $\exists i, k \in F_2 - F_1$  and  $\exists k \in (V - F_2 - F_1)$  such that  $(i, j)_k \in C$ .

Following this definition, it is shown that the local diagnosability  $t_l(x)$  of a node  $x \in V(G)$  in a system  $G(V, E)$  is the maximum number of  $t$  for  $G$  being locally  $t$ -diagnosable at  $x$ , that is,

$$t_l(x) = \max\{t \mid G \text{ is locally } t\text{-diagnosable at } x\}.$$

The authors show that there exists a relationship between local  $t$ -diagnosability at node  $x$  and the traditional  $t$ -diagnosability, presented as: a system  $G(V, E)$  is  $t$ -diagnosable if and only if  $G$  is locally  $t$ -diagnosable at  $x$ , for every  $x \in V(G)$ . Furthermore, the authors prove that a system  $G(V, E)$  is  $t$ -diagnosable if and only if  $\min\{t_l(x) \mid \forall x \in V(G)\} = t$ .

Recently in [2009] Chiang and Tan defined a local structure called extended star which is used to efficiently compute the node diagnosability under the MM\* model. An extended star, denoted as  $ES(x; n)$  of order  $n$  at node  $x$ , is defined as follows. Let  $x$  be a node in a graph  $G(V, E)$ .  $ES(x; n) = (V(x; n), E(x; n))$ , where the set of nodes  $V(x; n) = \{x\} \cup \{v_{ij} \in V \mid 1 \leq i \leq n, 1 \leq j \leq 4\}$ , and the set of edges  $E(x; n) = \{(x, v_{k1}), (v_{k1}, v_{k2}), (v_{k2}, v_{k3}), (v_{k3}, v_{k4}) \mid 1 \leq k \leq n\}$ . In other words, an extended star of order  $n$  at node  $x$   $ES(x; n)$ , implies that there exists  $n$  node-disjoint paths of length four starting from node  $x$  in the system. An example that shows a node  $x$  connected in an extended star structure is presented in Figure 17.

The authors prove that the node diagnosability of a node  $x$  is at least  $n$  if there exists an extended star  $ES(x; n) \subseteq G$  at node  $x$ . They also present an algorithm to diagnose the system provided that there is an extended star structure at each node. The algorithm has time complexity  $O(N\Delta)$ , where  $\Delta$  is the maximum degree of a node in the system.

## 9. COMPARISON-BASED DIAGNOSIS FOR STAR GRAPHS

The *star graph* is another topology for interconnection networks which has been used to deploy fault-tolerant multicomputer systems [Kavianpour 1996]. The diagnosability of star graphs under the MM\* model is presented by Zheng et al. [2002]. The diagnosis is also based on Maeng and Malek's [1981] comparison-based diagnosis model.

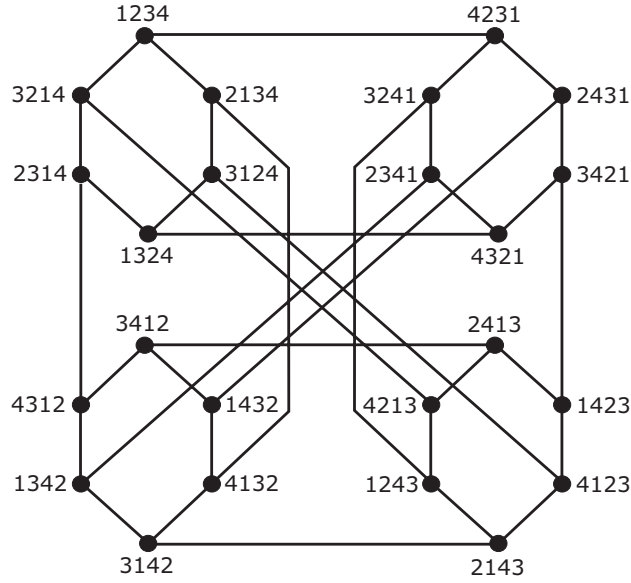


Fig. 18. An example of a four-dimensional star graph,  $S_4$ .

An  $n$ -dimensional star graph, also referred to as an  $n$ -star or  $S_n$ , is an undirected graph consisting of  $n!$  nodes and  $(n-1)n!/2$  edges [Akers and Krishnamurthy 1989]. Each node is uniquely assigned a label  $a_1a_2\dots a_m\dots a_n$ , which is a distinct permutation of the set of symbols  $\{a_1, a_2, \dots, a_m\}$ . Without loss of generality, let symbol set  $\{a_1, a_2, \dots, a_n\}$  be the set of integers  $\{1, 2, \dots, n\}$ . One node is linked by an edge to another node if and only if the label of one node can be obtained from the label of another node by interchanging the first symbol with the  $i$ th symbol, for  $2 \leq i \leq n$ . In  $S_n$  each node is connected to  $n-1$  nodes, that is, each node has degree  $n-1$ . Furthermore, each  $S_n$  can be decomposed into  $n$  star graphs, each of which  $(n-1)$ -dimensional.

For example, in a 4-star containing  $4!$  nodes, two nodes  $x$  with label 1234 and  $y$  with label 4231 are neighbors and joined through an edge. A 4-star graph ( $S_4$ ) is shown as an example in Figure 18.

Zheng et al. [2002] use the three sufficient conditions given in Sengupta and Dahbura's characterization [1992] and show that a system with  $N$  nodes is  $t$  diagnosable if: (1)  $N \geq 2t + 1$ ; (2) each node has degree at least  $t$ ; (3) for each  $X \subset V$  such that  $|X| = N - 2t + p$  and  $0 \leq p \leq t - 1$  then  $|T(X)| > p$ .

The authors also prove that an  $n$ -dimensional star graph is  $(n-1)$ -diagnosable under the comparison-based model for  $n \geq 4$ . To prove it, they show that  $S_n$  satisfies all three sufficient diagnosability conditions for  $n \geq 4$  as follows. The first condition: as the number of nodes  $N$  in  $S_n$  is  $n!$ , then  $n! \geq 2(n-1) + 1$  is true when  $n \geq 3$ . The second condition follows from the fact that each node of  $S_n$  has degree  $n-1$ . They show the third condition in two steps: first they prove, by contradiction, that for  $p = n-2$ , for an arbitrary  $X \subset V$  such that  $|X| = n! - 2(n-2) + p$  where  $0 \leq p \leq n-2$ , then  $|T(X)| > p$  is true; then they prove, also by contradiction, that for  $p = 0, 1, \dots, n-3$  then  $|T(X)| > p$  is true.

Finally, either of the polynomial algorithms presented in Sengupta and Dahbura [1992] and Yang and Tang [2007] can be applied to the  $n$ -dimensional star graphs to find the set of faulty nodes of the system if the number of faulty nodes is not greater than  $n-1$ .

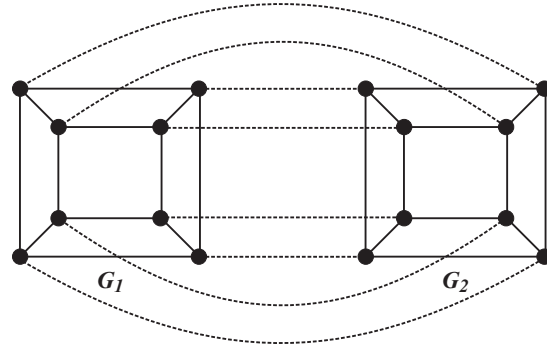


Fig. 19. An example of an  $MCN_3, G(G_1, G_2; L)$ .

## 10. COMPARISON-BASED DIAGNOSIS FOR MATCHING COMPOSITION NETWORKS

The diagnosability of *matching composition* networks is presented by Lai et al. [2004] and is also based on Maeng and Malek's [1981] comparison-based diagnosis model. A Matching Composition Network (MCN) is a network topology that consists of two components that are connected by a perfect matching. An MCN includes many topologies as special cases, such as the hypercube, the crossed cube, the twisted cube, and the Möbius cube [Cull and Larson 1995; Fan 1998]. MCNs can be recursively constructed. They are constructed from two graphs with the same number of nodes by adding a perfect matching between the nodes of the two graphs.

An MCN is a graph  $G = (V, E)$  defined as follows. Let  $G_1$  and  $G_2$  be two graphs with the same number of nodes and every node  $v$  in  $G_i$  has  $degree_{G_i}(v) \geq t$ , where  $i = 1, 2$ . Let  $L$  be an arbitrary perfect matching between the nodes of  $G_1$  and  $G_2$ , that is,  $L$  is a set of edges connecting nodes of  $G_1$  to nodes in  $G_2$  in a one-to-one manner. The resulting composition graph is an MCN; graphs  $G_1$  and  $G_2$  are called the *components* of the MCN.

Let  $MCN_i$  denote an  $i$ -dimensional MCN.  $MCN_1$  is a complete graph with two vertices. For  $n \geq 2$ , each  $MCN_n$  consists of two  $MCN_{n-1}$ , denoted by  $MCN_{n-1}^a$  and  $MCN_{n-1}^b$  with an arbitrary and perfect matching  $L$ .  $L$  is a set of edges connect  $MCN_{n-1}^a$  and  $MCN_{n-1}^b$ . The number of vertices in a  $MCN_n$  is  $2^n$  and each one has  $n$  neighbor vertices.

An MCN is represented by  $G(G_1, G_2; L)$  which has node set

$$V(G(G_1, G_2; L)) = V(G_1) \cup V(G_2)$$

and edge set

$$E(G(G_1, G_2; L)) = E(G_1) \cup E(G_2) \cup L.$$

An example of an  $MCN_3, G(G_1, G_2; L)$  is shown in Figure 19.

Lai et al. evaluate the diagnosability of matching composition networks under the MM\* model [Lai et al. 2004]. In their model  $M = (V, C)$  is also the comparison multigraph, and graph  $G$  represents the MCN. The notation  $(u, v)_w$  also represents a comparison, that is, node  $w$  compares the task outputs produced by nodes  $u$  and  $v$ . Let  $U \in V$  and  $\bar{U} = V - U$ ,  $T(G, U)$  is the set  $\{v \mid (u, v)_w \in C \text{ and } w, u \in U \text{ and } v \in \bar{U}\}$ . They show that an MCN  $G$  with  $N$  nodes is  $t$ -diagnosable if:

- (1)  $N \geq 2t + 1$ ;
- (2)  $degree_G(v) \geq t$  for every node  $v$  in  $G$ ;
- (3) for any two distinct subsets  $S_1, S_2 \in V(G)$  such that  $|S_1| = |S_2| = t$ , one of the following conditions are satisfied:

- (a)  $|T(G, U)| > p$ , where  $U = V - (S_1 \cup S_2)$ , and  $|S_1 \cap S_2| = p$ , or
- (b)  $\exists i, j \in S_1 - S_2$  and  $\exists k \in V - S_1 - S_2$  such that  $(i, j)_k \in C$ , or
- (c)  $\exists i, j \in S_2 - S_1$  and  $\exists k \in V - S_1 - S_2$  such that  $(i, j)_k \in C$ .

The authors also prove that an MCN  $G(G_1, G_2; L)$  – such that  $t \geq 2$ ,  $G_1$  and  $G_2$  are two graphs with the same number of nodes  $N$ ,  $N \geq t+2$ , every node  $v$  in  $G_i$  has  $degree_{G_i} \geq t$ , where  $i = 1, 2$  – is  $(t+1)$ -diagnosable. They also prove that the diagnosability of an  $n$ -dimensional hypercube, a crossed cube, a twisted cube, and the Möbius cube are  $n$  for  $n \geq 4$ .

Araki and Shibata introduce  $(t, k)$ -diagnosis in [2003]. The  $(t, k)$ -diagnosis model is a generalization of the PMC model [Preparata et al. 1967] and of the BGM [Barsi et al. 1976] model.  $(t, k)$ -diagnosis guarantees that at least  $k$  faulty units in a system are identified and repaired in each iteration provided that the number of faulty units does not exceed  $t$ , where  $k \leq t$ . Thus  $(t, k)$ -diagnosis allows correct but incomplete diagnosis.  $(t, k)$ -diagnosis is a generalization that also includes both one-step and sequentially diagnosable systems: in one-step diagnosis  $t = k$  and in sequential diagnosis  $k = 1$ .

Chang et al. in [2007] applied  $(t, k)$ -diagnosis for matching composition networks under the MM\* model. They prove that an MCN of  $n$  dimensions is  $(\Omega(\frac{2^{n \log n}}{n}), n)$ -diagnosable, for  $n > 5$ . They extend their result and prove that hypercubes, crossed cubes, twisted cubes, and Möbius cubes of  $n$  dimensions are all  $(\Omega(\frac{2^{n \log n}}{n}), n)$ -diagnosable, for  $n > 5$ . In Chang et al. [2007] the authors also present a polynomial-time  $O(|E|)$  algorithm for  $(t, k)$ -diagnosis under the MM\* model.

## 11. COMPARISON-BASED DIAGNOSIS FOR $T$ -CONNECTED AND PRODUCT NETWORKS

The diagnosability of  $t$ -connected networks and product networks under the comparison-based diagnosis was presented by Chang et al. in [2004] also under the MM\* model. A graph  $G$  is  $t$ -connected if  $\kappa(G) \geq t$  where  $\kappa(G) = \min\{|V'| \text{ such that } V' \subseteq V \text{ and } G - V' \text{ is not connected}\}$ .

A product network is generated by applying the graph Cartesian product operation to factor networks. A Cartesian product network  $G = G_1 \times G_2$  [Araki and Shibata 2000] of two graphs  $G_1 = (V_1, E_1)$  and  $G_2 = (V_2, E_2)$  is the graph  $G = (V, E)$ . The graphs  $G_1$  and  $G_2$  are called the *factors* or *component networks* of graph  $G$ . The set of nodes  $V$  and the set of edges  $E$  of  $G$  are given by:

- (1)  $V = \{(x, y) \mid x \in V_1 \text{ and } y \in V_2\}$ , and
- (2) for  $u = (x_u, y_u)$  and  $v = (x_v, y_v)$  in  $V$ ,  $(u, v) \in E$  if and only if  $(x_u, x_v) \in E_1$  and  $y_u = y_v$ , or  $(y_u, y_v) \in E_2$  and  $x_u = x_v$ .

As an example, Figure 20 shows two network graphs  $G_1$  and  $G_2$  and the corresponding Cartesian product network graph  $G_1 \times G_2$ .

Chang et al. [2004] evaluate the diagnosability of these topologies also assuming the conditions given by Sengupta and Dahbura [1992]. They show that a  $t$ -regular and  $t$ -connected network with  $N$  nodes and  $t > 2$  is  $t$ -diagnosable if  $N \geq 2t + 3$ . Moreover, the product network of  $G_1$  and  $G_2$  is shown to be  $(t_1 + t_2)$ -diagnosable, where  $G_i$  is  $t_i$ -connected with regularity  $t_i$  for  $i = 1, 2$ .

## 12. STRONG DIAGNOSABILITY FOR COMPARISON-BASED DIAGNOSIS

The strong  $t$ -diagnosability of a system under the PMC model was first presented by Lai et al. in [2005]. A system is *strongly  $t$ -diagnosable* if it is  $(t+1)$ -diagnosable and there is no node such that all its neighbors are faulty. In other words: strong diagnosability

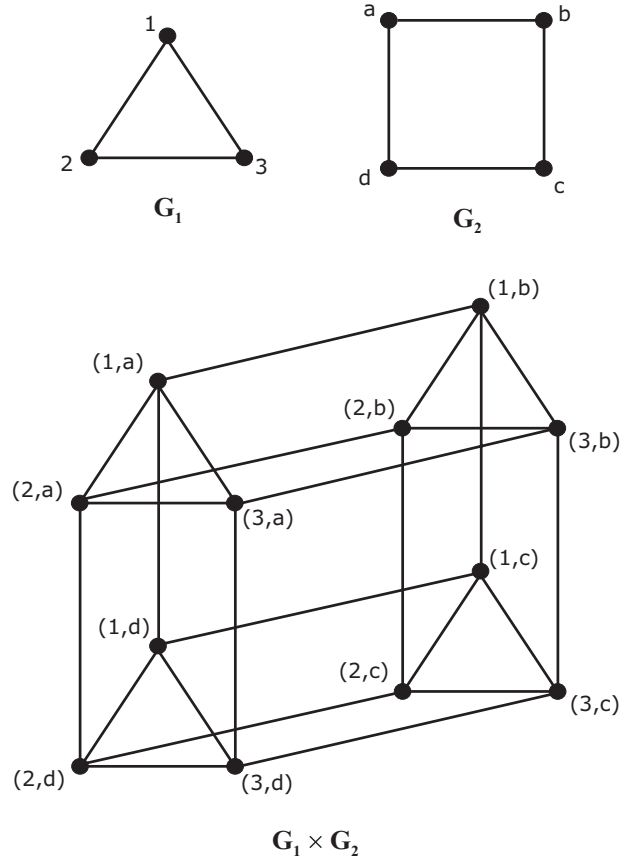


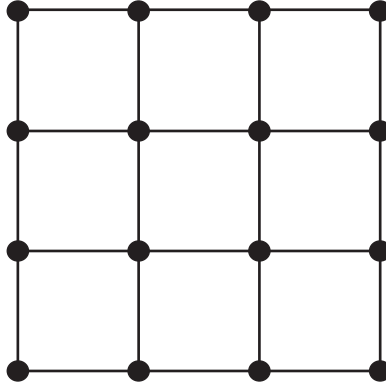
Fig. 20. Two networks  $G_1$  and  $G_2$  and the corresponding product network  $G_1 \times G_2$ .

shows the ability of a  $t$ -diagnosable system to detect an extra faulty node assuming that all neighbors of any node do not become simultaneously faulty.

Sheu et al. [2008] were the first to investigate the strong diagnosability of systems under the  $MM^*$  model. They show that a  $t$ -regular and  $t$ -connected network in which  $N \geq 2t + 6$  and  $t \geq 4$  is strongly  $t$ -diagnosable if the system is triangle free and the intersection of the sets of neighbors of any two nodes in the system has at most  $t - 2$  nodes.

Hsieh and Chen [2008a] investigated the strong diagnosability of a class of product networks under the  $MM^*$  model. As defined in Section 11, a product network is generated by applying the graph Cartesian product operation to factor networks. Product networks include topologies such as the hypercubes, mesh-connected  $k$ -ary  $n$ -cubes, torus-connected  $k$ -ary  $n$ -cubes, and hyper-Petersen networks. Regular product networks can be classified in two subclasses: homogeneous product networks and heterogeneous product networks. *Homogeneous product networks* refer to every factor network of the product that is  $t$ -diagnosable and  $t$ -regular, while *heterogeneous product networks* are comprised of two different factor networks, one of which is  $t$ -diagnosable and the other is  $t$ -connected.

For  $t_i > 3$ , the strong diagnosability of homogeneous product networks  $G_1 \times G_2 \times \dots \times G_k = t_1 + t_2 + \dots + t_k$ , where  $G_i = (V_i, E_i)$  is a  $t_i$ -diagnosable and  $t_i$ -regular network with  $N_i$

Fig. 21. An example of  $M_4^2$ .

nodes, and  $i = 1, 2, \dots, k$ . Consider that  $G_i = (V_i, E_i)$  is a  $t_i$ -diagnosable and  $t_i$ -regular network with  $N_i$  nodes for  $i = 1, \dots, m$  and let  $G_j = (V_j, E_j)$  be a  $t_j$ -connected and  $t_j$ -regular network with  $N_j \geq 2t_j + 1$  nodes for  $j = m + 1, \dots, k$ . For  $t_i > 3$ , if  $G = G_1 \times G_2 \times \dots \times G_k$ , then the strong diagnosability of  $G$  is  $t_1 + t_2 + \dots + t_k$ . For the strong diagnosability of nonregular product networks, consider that  $G_1 = (V_1, E_1)$  is  $t_1$ -diagnosable,  $L_{k_i}$  be a  $k_i$ -node linear array, and  $k_i \geq 2$  for  $1 \leq i \leq l$ . The authors prove that, for  $t_i > 3$ , the nonregular product network  $G = G_1 \times L_{k_1} \times L_{k_2} \times \dots \times L_{k_l}$  is strongly  $(t_1 + l)$ -diagnosable.

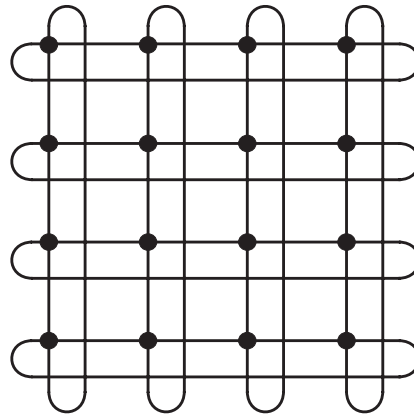
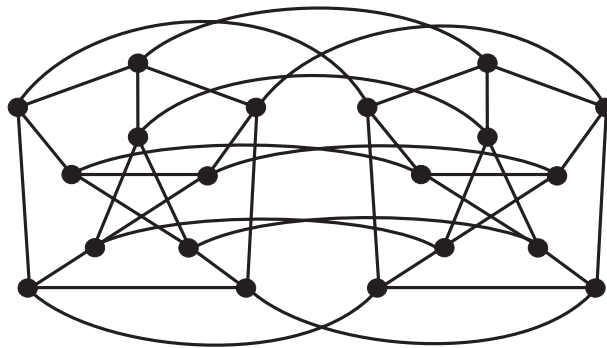
The strong  $t$ -diagnosability of four different product network topologies, all of which are  $t$ -regular and  $t$ -connected, is shown in Hsieh and Chen [2008a]: the  $n$ -dimensional hypercube, the mesh-connected  $k$ -ary  $n$ -cube, the torus-connected  $k$ -ary  $n$ -cube, and finally the  $n$ -dimensional hyper-Petersen network. For all of these networks,  $N \geq 2t + 1$  nodes, where  $t > 2$ ; each node  $v$  of  $G$  has degree of at least  $t$ . The first strong diagnosability result is for the  $n$ -dimensional hypercube, which is  $n$  for  $n \geq 5$ . The other three topologies and their strong diagnosability results are presented next.

A mesh-connected  $k$ -ary  $n$ -cube [Bettayeb 1995], denoted by  $M_k^n$ , is recursively defined as follows: let  $L_k$  be a  $k$ -node linear array, (1)  $M_k^1 = L_k$ , for  $k \geq 2$ , and (2)  $M_k^n = M_k^{n-1} \times L_k$  for  $n \geq 2$ . A  $k$ -ary  $n$ -cube  $M_k^n$  has  $k^n$  nodes. As an example, Figure 21 shows  $M_4^2$ . The authors prove that the strong diagnosability of  $M_k^n = n$  for  $n \geq 5$ .

A torus-connected  $k$ -ary  $n$ -cube [Bettayeb 1995], denoted by  $T_k^n$ , is recursively defined as follows: let  $R_k$  be a ring (cycle) of length  $k$ , where  $k \geq 3$ . Then, (1)  $T_k^1 = R_k$ , and (2)  $T_k^n = T_k^{n-1} \times R_k$  for  $n \geq 2$ . A torus-connected  $k$ -ary  $n$ -cube  $T_k^n$  also has  $k^n$  nodes. Figure 22 shows an example of  $T_4^2$ . The strong diagnosability of a torus-connected  $k$ -ary  $n$ -cube is  $2n$  for  $k \geq 3$  and  $n \geq 4$ .

A  $n$ -dimensional hyper-Petersen network [Das et al. 1995], denoted by  $HP_n$  for  $n \geq 3$ , is defined as  $HP_n = P \times Q_{n-3}$ , where  $P$  is a Petersen graph. An  $HP_n$  is  $n$ -connected and  $n$ -regular and has  $10 * 2^{n-3}$  nodes. Figure 23 shows an example of  $HP_4$ . The strong diagnosability of a  $HP_n = n$  for  $n \geq 5$ .

Later Hsieh and Chen [2008b] presented the strong diagnosability for a class of Matching Composition Networks (MCNs) under the  $MM^*$  model. They evaluated the strong diagnosability of  $n$ -dimensional crossed cubes, Möbius cubes, twisted cubes, and locally twisted cubes. An  $n$ -dimensional crossed cube  $CQ_n$  is strongly  $n$ -diagnosable for  $n \geq 5$ . An  $n$ -dimensional Möbius cube  $MQ_n$  is strongly  $n$ -diagnosable for  $n \geq 5$ . An  $n$ -dimensional twisted cube  $TQ_n$  is strongly  $n$ -diagnosable for an odd integer  $n \geq 5$ . Finally, an  $n$ -dimensional locally twisted cube  $LTQ_n$  is strongly  $n$ -diagnosable for  $n \geq 4$ .

Fig. 22. An example of  $T_4^2$ .Fig. 23. An example of  $HP_4$ .

### 13. THE BROADCAST COMPARISON DIAGNOSIS MODEL

The broadcast comparison diagnosis model was introduced by Blough and Brown in [1999]. This model applies distributed diagnosis based on the  $MM^*$  comparison model [Maeng and Malek 1981] for systems that have a weak reliable broadcast [Hadzilacos and Toueg 1993] service available. In this model, a distributed diagnosis procedure is used which is also based on comparisons of redundant task outputs.

The system is also modeled as a graph  $G(V, E)$ . Tasks are assigned to pairs of different nodes. These two nodes execute the task and the task's outputs are sent to all nodes using reliable broadcast. After task outputs are received, they are compared in order to detect faults. The comparisons are performed by all nodes of the system. Figure 24 shows this procedure. In this figure node 1 sends the same task to node 2 and to node 3 which execute the task and broadcast their tasks' outputs to all system nodes.

All fault-free nodes in the system compare the two produced outputs, including the nodes that produced the task outputs. The syndrome is the complete collection of all comparison outcomes. As soon as each node executes all comparisons, it completes the diagnosis of the system assuming itself as fault free.

The main assumptions of the broadcast comparison model are as follows.



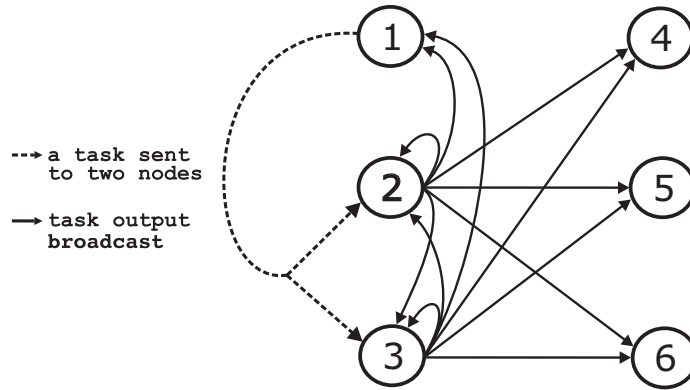


Fig. 24. A task is sent from node 1 to nodes 2 and 3. The task outputs are broadcast to the whole system.

- (1) When two fault-free nodes execute the same task, they produce the same output, and the comparison of these outputs executed by all fault-free nodes of the system indicates a match.
- (2) A faulty node always produces a task output that results in a mismatch when compared with the outputs produced by any other faulty or fault-free node.
- (3) Any message broadcast from a fault-free processor is correctly received by all other fault-free processors in a bounded time.
- (4) The time for any task to produce an output is bounded.
- (5) Each processor has a unique identifier.
- (6) Fault-free processors can correctly identify the sender of a broadcast message.
- (7) Values sent by faulty processors are correctly received by fault-free processors, furthermore a fault-free processor comparing a faulty processor and any other processor always produces a mismatch.

Assumptions (1) and (2) are inherited from MM and MM\* models. The other assumptions are made in order to guarantee these two assumptions. Assumption (3) is the basic assumption of weak reliable broadcast. Weak reliable broadcast [Hadzilacos and Toueg 1993] requires that fault-free processors agree on all messages, even those sent by faulty processors, but it does not have any message ordering requirements. Assumption (7) prevents values from being modified during communication.

Blough and Brown [1999] give a polynomial-time algorithm for computing the diagnosability of the system under the broadcast comparison model. Five definitions are necessary to characterize the diagnosability of the system.

- (1) An independent set in graph  $G(V, E)$  is a subset  $V' \subseteq V$  such that for all  $u, v \in V'$ ,  $(u, v) \notin E$ .
- (2) For graph  $G(V, E)$  and processor  $u \in V$ ,  $N(u) = \{v \in V \mid (u, v) \in E\}$ , that is, the neighbor set of processor  $u$ . Also,  $|N(u)| = d(u)$ .
- (3) For graph  $G(V, E)$  and set  $Z \subseteq V$ ,  $N(Z) = \{v \in V - E \mid \exists u \in Z \text{ and } (u, v) \in E\}$ , that is, the neighbor set of  $Z$ .
- (4) For graph  $G(V, E)$ ,  $P_G$  is the set of partitions of  $V$  into four pairwise disjoint sets  $(X, Y, Z_1, Z_2)$  such that: (1)  $X \neq \emptyset$ ; (2)  $N(X) \subseteq Y$ ; (3)  $Z_1 \cup Z_2 \neq \emptyset$ ; and (4)  $Z_1$  and  $Z_2$  are independent sets.
- (5) For graph  $G(V, E)$ ,  $\kappa$  is a function from  $P_G$  to the set of positive integers such that for all  $p = (X, Y, Z_1, Z_2) \in P_G$ ,  $\kappa(p) = |Y| + \max(|Z_1|, |Z_2|)$ .

A system  $G(V, E)$  is  $t$ -diagnosable if and only if for all  $p \in P_G$ ,  $\kappa(p) > t$ .

```

Algorithm: Static-Complete
/* Input: The syndrome of the system and the diagnosability  $t$  */
/* Output: The set  $FF$  (fault-free) and the set  $F$  (faulty) */
1)  $F \leftarrow \emptyset$ ;  $FF \leftarrow \{x\}$ ;
2) for each edge  $(u, v)$  with a comparison outcome 0
    $FF \leftarrow FF \cup \{u, v\}$ ;
3) if  $|V| - |FF| \leq t$  then
    $F \leftarrow V - FF$ ; stop;
4) for each edge  $(u, v)$  with a comparison outcome 1
   if  $u \in FF$  then  $F \leftarrow F \cup \{v\}$ ;
   if  $v \in FF$  then  $F \leftarrow F \cup \{u\}$ ;
5) if  $|F| < t$  then
    $F \leftarrow F \cup Find\_Remaining(G[V - FF - F], t - |F|)$ ;
6)  $FF \leftarrow V - F$ ;

```

Fig. 25. Algorithm Static-Complete.

The diagnosability of a system of  $N$  processors given a complete comparison graph is  $N - 1$ . The diagnosability of a system that does not have a complete comparison graph available is either  $d_{min}(G)$  or  $d_{min}(G) - 1$ , where the degree  $d(u)$  of a processor  $u$  in  $G(V, E)$  is the number of edges of  $G(V, E)$  incident on  $u$ . The minimum degree of a system  $G(V, E)$  is  $d_{min}(G) = \min_{u \in V} d(u)$ .

In [1999] Blough and Brown also present polynomial-time algorithms to diagnose static and dynamic fault situations using the broadcast comparison model.

In a static fault situation no faults occur in the system from the time the comparisons begin until diagnosis is complete. Blough and Brown [1999] present algorithm Static-Complete for the diagnosis of a system under the static fault situation, given a complete syndrome.

Figure 25 shows algorithm Static-Complete. The algorithm runs at each system node  $x$  and receives as input the syndrome of the system and the diagnosability  $t$ . Every node assumes itself as fault free, adding itself to the set of fault-free nodes  $FF$ ; this is represented in step 1. In step 2, any processor that has a comparison outcome that indicates a match is also added to set  $FF$ . If the number of remaining processors (not in  $FF$ ) is at most  $t$ , step 3 stops the algorithm. If this is not the case, step 4 identifies faulty processors in  $FF$  and adds them to set  $F$ . Step 5 determines if there exists some faulty processors that remain unknown and adds them to set  $F$  using the *Find.Remaining* function that is show in Figure 26. Finally, in step 6 the algorithm finishes in which set  $FF$  is obtained.

In some situations where the number of faulty nodes is much smaller than  $t$ , it is still possible for fault-free processors to correctly diagnose the system without performing all comparisons; this situation is referred to as diagnosis with a partial syndrome. Algorithm Static-Partial is given for situations in which only a partial syndrome is available. In these cases no algorithm is guaranteed to diagnose the status of all processors, that is, the diagnosis is guaranteed to be correct but may be incomplete.

As in real systems faults can occur during the execution of the diagnosis algorithm, Blough and Brown [1999] present algorithm Dynamic to diagnose systems under a dynamic fault situation. Nevertheless, they assume that once a processor becomes faulty, it remains faulty until the next execution of the diagnosis algorithm. Furthermore, this model allows fault-free processors to become faulty, while it is not allowed for faulty processors to become fault free during the diagnosis execution. Task outputs are timestamped before they are broadcast and the clock of fault-free processors must advance at an approximately correct and bounded rate.

```

Function: Find_Remaining
/* Input: A graph  $\hat{G}(\hat{V}, \hat{E})$  and an integer  $\hat{t}$  where  $0 < \hat{t} < |\hat{V}|$  */
/* Output: A faulty set  $\hat{F}\hat{F}$  */
1) for each  $u \in \hat{V}$  with a comparison outcome 0
2)   if  $|N(u)| = \hat{t}$  then
3)      $\hat{F}\hat{F} \leftarrow \{u\}$ ;
4)     for each  $v \in \hat{V} - N(u) - \{u\}$ 
5)       if  $N(v) = N(u)$  then  $\hat{F}\hat{F} \leftarrow \hat{F}\hat{F} \cup \{v\}$ ;
6)   if  $|\hat{F}\hat{F}| = |\hat{V}| - \hat{t}$  then return  $N(u)$ ;

```

Fig. 26. Function Find\_Remaining.

The main difference of the broadcast comparison model and the MM\* model is that the broadcast comparison model is fully distributed while the MM\* model relies on a central observer that receives task outputs and executes all comparisons. In the broadcast comparison model all fault-free processors produce the same set of comparison outcomes.

The main purpose of the broadcast comparison model is to reduce the latency and the time one node must remain in a given state, not the number of tests or comparisons executed. The system must present a built-in primitive (for instance, implemented in hardware) equivalent to weak reliable broadcast. The system was implemented in the COmmon Spaceborne Multicomputer Operating System (COSMOS). They also shown results obtained with a simulator for the JPL MAX multicomputer system running COSMOS.

#### 14. GENERALIZED MODELS FOR DISTRIBUTED COMPARISON-BASED DIAGNOSIS

The Generalized Distributed Comparison-based (GDC) model [Albini et al. 2005; Albini and Duarte Jr. 2001] assumes a fully connected system  $S$  also represented by a graph  $G = (V, E)$ , in which  $\forall i \in V$  and  $\forall j \in V$ ,  $\exists (i, j) \in E$ . This model incorporates all the assumptions of the MM model, plus one: the time for a fault-free node to produce an output for a task is bounded. This model is fully distributed, that is, fault-free nodes both execute the comparisons and diagnose the system based on the comparison syndrome. Although in this sense the broadcast comparison model is also fully distributed, the generalized distributed comparison-based model does not assume a reliable broadcast system primitive; reliable unicast is enough to implement the model.

The GDC model defines a multigraph,  $M(S)$ , to represent the way tests are executed in the system.  $M(S)$  is a directed multigraph defined over graph  $G$ , when all nodes of the system are fault free. Nodes of the system can be either faulty or fault free. A node becomes faulty by either crashing or by replying arbitrarily to a given query. A change of the state of a node is called an *event*. The states of the nodes are also determined by comparing the task output of one node with the output generated by another node for the same task. As the MM model is assumed, a fault-free node comparing outputs produced by a faulty node and any other faulty or fault-free node always produces a mismatch.

In Albini et al. [2005] and Albini and Duarte Jr. [2001] a hierarchical comparison-based adaptive distributed system-level diagnosis algorithm—called *Hi-Comp*—is presented based on the generalized model. The algorithm is distributed, that is, it runs in every node of the system and every node performs the complete diagnosis. A *testing round* is defined as the interval of time that all fault-free nodes need to diagnose all nodes of the system. An assumption is made that after node  $i$  tests node  $j$  in a certain testing round, node  $j$  cannot be affected by a new event in this testing round.

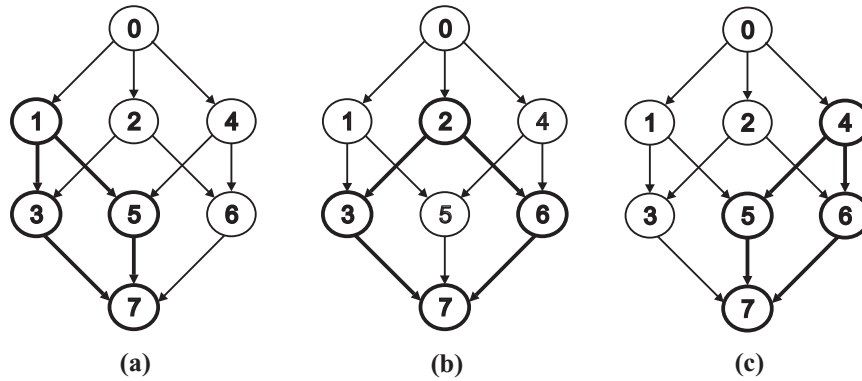


Fig. 27.  $T_0(S)$ : node 0 obtains diagnostic information from nodes 1, 2, and 4.

The algorithm employs a testing strategy represented by a graph  $T(S)$  which is a virtual hypercube when the number of nodes is a power of two. Nodes have sequential identifiers  $(0..N-1)$ , and each node can thus compute its set of neighbors in  $T(S)$ . The *diagnostic distance* between node  $i$  and node  $j$ ,  $d_{i,j}$ , is defined as the shortest distance between node  $i$  and node  $j$  in  $T(S)$ . For example, in Figure 27 the diagnostic distance between node 0 and node 2 is 1.

A graph  $T_i(S)$  is defined as a directed graph based on  $T(S)$  and maintained by node  $i$  that shows how nodes obtain diagnostic information. Figure 27 shows  $T_0(S)$  for a system of 8 nodes; node 0 obtains diagnostic information about: (a) nodes [3, 5, 7] from node 1, (b) nodes [3, 6, 7] from node 2, and (c) nodes [5, 6, 7] from node 4.

In each testing round, a node  $i$  running Hi-Comp initially tests pairs of sons in  $T_i(S)$ . When the comparison of two distinct nodes  $p$  and  $q$  indicates a match, node  $i$  classifies the two tested nodes as fault free. Otherwise, if the comparison indicates a mismatch, the tested nodes are classified as *undefined*. If node  $i$  had already tested a pair of nodes as fault free, now it compares one of those fault-free nodes with each of the two undefined nodes. On the other hand, if node  $i$  has not yet diagnosed any fault-free node, the two nodes remain undefined. If after node  $i$  tests all its sons it does not identify any fault-free node, that is, all sons of node  $i$  are classified as undefined, node  $i$  proceeds to test the sons of its sons, and so on until a comparison indicates a match, or node  $i$  tests all nodes in  $T_i(S)$ .

As soon as node  $i$  classifies any node  $p$  as fault free, node  $i$  obtains from node  $p$  diagnostic information about every node  $k \in V \mid d_{i,k} \leq d_{i,p} + d_{p,k}$ . Node  $i$  can obtain diagnostic information about a node  $j$  through more than one node. As an example, in Figure 27 node 0 can obtain diagnostic information about node 3 from node 1 or node 2. To assure that node  $i$  has the most recent diagnostic information about node  $j$  the algorithm employs *timestamps*, implemented as event counters [Duarte Jr. et al. 2000].

The latency of Hi-Comp is proved to be  $\log_2 N$  testing rounds, the maximum number of tests executed is  $O(N^3)$ , and the algorithm is  $(N-1)$ -diagnosable.

Another general hierarchical comparison-based adaptive distributed diagnosis (GDC\*) model was proposed in Ziwich et al. [2005]. In this model a fault-free node tests other nodes and based on test results, classifies the tested nodes in sets. A test is also performed by sending a task to two nodes. Task outputs are then compared; if the comparison produces a match, the two nodes are classified in the same set. On the other hand, if the comparison results in a mismatch, the two nodes are classified in different sets, according to their task results. One of the sets contains all fault-free

nodes. If nodes are classified in more than one set, then there are faulty nodes in the system.

The GDC\* model identifies crashed nodes, and also identified nodes that have not crashed but do not reply the correct and expected data for the tasks. The following assumptions are made about the system.

- (1) A fault-free node comparing outputs produced by two fault-free nodes always produces a match.
- (2) A fault-free node comparing outputs produced by a faulty node and a fault-free node always produces a mismatch.
- (3) The time interval required for a fault-free node to produce an output for a task is bounded.

The proposed model is the first distributed comparison-based model that allows the task outputs of two faulty nodes to be equal to each other, an early assumption of Chwa and Hakimi's model [Chwa and Hakimi 1981b].

In Ziwich et al. [2005] the authors also propose an algorithm for the GDC\* model called *Hi-Dif*. The algorithm also employs a testing strategy represented by a graph  $T(S)$  that is a virtual hypercube. The algorithm identifies crashed nodes in the system and also classifies nodes in sets. These sets allow the identification of which nodes are returning a given output for the input task. With these sets it is possible to identify fault-free nodes, crashed nodes, and the faulty nodes that returned different outputs. The latency of Hi-Dif is proved to be  $\log_2 N$  testing rounds, the maximum number of tests executed is  $O(N^2)$ , and the algorithm is  $(N - 1)$ -diagnosable.

The generalized distributed comparison-based models have been applied for detecting nonauthorized modifications of replicated data available in the Web [Ziwich et al. 2005]. A related work by Martins et al. [2006a, 2006b] applies distributed comparison-based diagnosis to tolerate manipulation attacks in computational grids. Comparisons performed on task results allow the detection and isolation of malicious nodes.

## 15. PROBABILISTIC COMPARISON-BASED MODELS

Probabilistic comparison-based models were first introduced by Dahbura et al. [1987]. All these models assume a fault probability, that is, the probability that a unit produces an incorrect output, and the diagnosability is computed with this probability. Thus, these models do not impose an upper bound on the number of faulty units in the system.

There are two basic probabilistic approaches for solving the diagnosis problem. These approaches were proposed for classical system-level diagnosis, and probabilistic comparison-based diagnosis appeared later. The first approach is to restrict diagnosis to a set of faulty units with a sufficiently high probability [Fujiwara and Kinoshita 1978; Maheshwari and Hakimi 1976]. The other approach is to perform diagnosis for the whole system, and this is then proved to be correct with a high probability [Blough et al. 1988, 1988; Blount 1977; Dahbura et al. 1987; Rangarajan and Fussell 1988]. In many cases these models reflect the actual fault environment in a more precise way, but they are often more difficult to analyze.

In the probabilistic comparison-based diagnosis model proposed by Dahbura et al. [1987], the system is also represented by a graph  $G = (V, E)$ . Tasks are also sent to pairs of units and the task outputs are compared to identify faulty units. The collection of all outcomes is also called the syndrome. The basic assumptions of this system follow.

- $m$  is the total number of possible distinct incorrect results which a faulty processor can produce for a task;
- $W_i \mid 1 \leq i \leq m$  is one of the  $m$  possible incorrect results for a task;

- $P(W_i)$  is the probability that a faulty unit produces the incorrect results  $W_i$  for a task; and,
- $p$  is the probability that a faulty unit produces a correct task output.

The following results are obtained from the evaluation of this model [Dahbura et al. 1987].

- (1) the probability  $P_{1,0}$  that the comparison of two outputs indicates a match is equal to  $p$  when one of the units that produced the output is faulty, and
- (2) the probability  $P_{2,0}$  that the comparison of two outputs indicates a match is equal to  $p^2 + P(W_1)^2 + \dots + P(W_m)^2$  when both units that produced the outputs are faulty.

The authors assume that the probability distribution for a unit to produce incorrect results is uniform; then  $\forall i, P(W_i) = (1 - p)/m$ . Thus the probability that two faulty units that have their outputs compared produce a match is  $P_{2,0} = p^2 + ((1 - p)^2/m)$ . Furthermore it is assumed that  $m$  is extremely large, thus  $P_{2,0} \approx p^2$ .

Another probabilistic and comparison-based diagnosis model was proposed by Pelc in [1991]. In this model, also called the  $(p, k)$ -probabilistic model, the same task with  $k$  possible outcomes are sent to the units. Each unit has the same probability  $p < 1/2$  to become faulty and failure of distinct units are independent. This model assumes that:

- fault-free units always give correct answers; and
- faulty units give independent random answers with uniform probability  $1/k$  each of them, but eventually the outcomes from two faulty units may match.

Like previous comparison-based models, the task outcomes are compared and the result, match (0) or mismatch (1), is then used to identify the faulty units in the system. The probability of a match being produced by the comparison of the outputs produced by two units, one fault free and the other faulty, or by two faulty units is  $q = 1/k$ . This is a difference of this model to the model proposed by Dahbura et al. [1987] in which the probability of obtaining an incorrect answer from a faulty processor is much smaller than that of the correct answer. Hence, in Dahbura's model the probability of obtaining a match when comparing two faulty units is  $q^2$  and the probability of obtaining a match when comparing a faulty unit and a fault-free unit is  $q$ .

A system is called diagnosable in this model if for any possible syndrome, there exists a unique most probable set of faulty units generating this syndrome. If this set exists, this is diagnosed as the faulty units in the system. Considering the  $(p, k)$ -probabilistic model, the authors have proved that:

- (1) a system with two units is not diagnosable;
- (2) assuming that  $p < 1/(k + 1)$ , an optimal diagnosable system with  $N > 2$  units has  $N - \lfloor N/3 \rfloor$  edges or connection links; and
- (3) the *diagnosis* and the *diagnosability* problems are NP-hard for general topology systems.

Blough and Pelc in [1992] present efficient polynomial-time diagnosis algorithms for Pelc's model [Pelc 1991], considering a large class of systems represented by bipartite graphs, which includes hypercubes, grids, and forests. They also show that optimal diagnosis for a general topology system is NP-hard. A linear-time algorithm to perform optimal diagnosis in a ring is also presented.

Another probabilistic comparison-based model is presented by Rangarajan and Fussell in [1988] which is based on the evaluation of multiple syndromes, instead of just one. In Fussell and Rangarajan [1989] the same authors propose an algorithm for this model, in which the probability of correct diagnosis approaches 1 when the number of tests performed on each processor is slightly higher than  $\log_2 N$ . In Lee and

Shin [1994] a provably optimal algorithm for the same model is presented. A solution for the probabilistic diagnosis of sparsely interconnected systems is presented in Choi and Jung [1990].

## 16. EVOLUTIONARY COMPARISON-BASED DIAGNOSIS

Evolutionary system-level diagnosis was first introduced by Elhadeif and Ayeb in [2000]. This theoretical work actually investigates how a Genetic Algorithm (GA) performs when applied to the problem of identifying faults given a system syndrome under the PMC model. Several other evolutionary algorithms were also implemented and compared for this task in Nassu et al. [2005]. Comparison diagnosis based on evolutionary computing was also introduced by Elhadeif and Ayeb in [2001a].

A genetic algorithm has the following components [Elhadeif and Ayeb 2001a]:

- (1) A representation of potential solutions to the problem, called a chromosome or individual. This is a binary array of length  $N$  that when used for system diagnosis represents which nodes are faulty and which are fault free. The chromosome is represented by  $\langle s_1 s_2 s_3 \dots s_N \rangle$  where  $s_i$  is the status of node  $u_i \in V$ . The status  $s_i$  of node  $u_i$ —also called a gene—can be 0 (fault free) or 1 (faulty). For example, for a system with 8 nodes, the chromosome  $v = (01000100)$  represents a potential solutions where node 2 and node 6 are faulty. A set of individuals is called a population.
- (2) A procedure to create an initial population of solutions.
- (3) An evaluation function which gives the fitness of each individual. The evaluation function can be seen as the probability that a potential solution is correct.
- (4) Genetic operators, which are employed to modify individuals within a population to produce new individuals. Genetic operators include, for instance, selection, crossover, and mutation, defined as follows. Selection forms a new generation by choosing those individuals from the old population that have the highest fitness. Crossover takes two individuals—called parents—and produces new individuals—called children—which inherit genetic material (bits) from their parents. Mutation toggles random bits within a population.
- (5) Parameters employed by the genetic algorithm, such as the population size  $P$  and the probabilities of applying genetic operators.

Each of these components has a direct impact on the solution obtained as well as the performance of the genetic algorithm. Elhadeif and Ayeb [2001a] present an GA-based algorithm—called Genetic-Comparison-Diagnosis—for system fault diagnosis under the comparison model. The algorithm is presented in Figure 28. The algorithm receives as input a graph  $G(V, E)$  and the comparison syndrome  $\sigma$  and produces as output the set of faulty nodes  $F$  and the set of fault-free nodes  $FF$ .

The fitness function of a chromosome  $v$ ,  $FT(v)$ , is given after the following definitions, which are necessary to understand it. Let  $N(u_i)$  be the neighborhood set of node  $u_i$ . Considering the multigraph  $M = (V, C)$ ,  $S_\sigma(u_i) = \{r((u_i, u_j)_{u_k}) \in \sigma \text{ such that } u_j \in N(u_i) \text{ and } (u_i, u_j)_{u_k} \in C\}$ . In other words,  $S_\sigma(u_i)$  is the subset of syndrome  $\sigma$  corresponding to comparisons between unit  $u_i$  and its neighbors  $N(u_i)$ . Considering chromosome  $v$ ,  $v[i]$  denotes the  $i^{\text{th}}$  bit of the binary array  $v$ , and  $\sigma^*$  denotes its corresponding comparison syndrome. The fitness value of node  $u_i$  is given by  $f(v[i])$ , that is,  $f(v[i])$  is the probability of status correctness of node  $u_i$ .

$$FT(v) = \frac{\sum_{i=1}^n f(v[i])}{N}, \text{ where } f(v[i]) = \frac{|S_\sigma(u_i) \cap S_{\sigma^*}(u_i)|}{|N(u_i)|}$$

```

Algorithm: Genetic-Comparison-Diagnosis
/* Input:  A graph  $G(V, E)$  and syndrome  $\sigma$  */
/* Output: A fault set  $F$  and a fault-free set  $FF$  */

begin
  Generate initial population of solutions  $Pop$ ;
  for each  $v \in Pop$  do
    compute  $FT(v)$ ;
  end for
   $Elite \leftarrow$  solution in  $Pop$  with the highest fitness;
  while ( $\forall v \in Pop, FT(v) \neq 1$ ) do
    Selection( $Pop$ );
    Mutation( $Pop$ );
    Crossover( $Pop$ );
    for each  $v \in Pop$  do
      compute  $FT(v)$ ;
    end for
    Elitism( $Pop, Elite$ );
  end while
   $F \leftarrow \mathcal{F}(v)$  such that  $v \in Pop$  and  $FT(v) = 1$ ;
   $FF \leftarrow V - F$ ;
end

```

Fig. 28. The Genetic-Comparison-Diagnosis algorithm.

This genetic algorithm has a slight modification compared to standard GAs [Elhadef and Ayeb 2001a]: the mutation process is performed before crossover. This is made because the mutation operator used in the Genetic-Comparison-Diagnosis algorithm is based on fitness values. In standard mutation process, each bit has an equal chance to suffer mutation. Instead, the authors consider each bit fitness value  $f(v[i])$  as its probability to be toggled. Hence, chromosomes should not suffer a crossover before the mutation. The authors present experimental results comparing the standard (equal chance) mutation operator to the new mutation process in system with nodes varying from 8 to 500 nodes. The results show that the algorithm under the new mutation process completes the diagnosis within fewer generations.

The Genetic-Comparison-Diagnosis algorithm uses the elitism strategy, that is, at the end of each iteration, the best chromosome is always compared with an elite chromosome—that is the best chromosome so far, and has a copy of it stored separately from the population. If the best chromosome is better than elite chromosome, a copy of it becomes the elite chromosome. On the other hand, if the best chromosome is not better than elite's one, a copy of the elite chromosome replaces the worst chromosome in the population. Elitism guarantees that the quality of the best solution found over generation is always increasing.

Finally,  $\mathcal{F}(v)$  denotes the set of faulty nodes according to the chromosome  $v$ , which are the nodes with a gene value equal to 1. In the fitness function  $FT$ , if the chromosome  $v$  corresponds to the optimal solution, that is,  $\mathcal{F}(v)$  is the set of all faulty nodes in the system, then  $FT(v) = 1$  and  $v$  is the system diagnosis.

The time complexity of the Genetic-Comparison-Diagnosis algorithm is  $O((|E| P \ln P^2) / \ln r)$  in the worst case and  $O((|E| P \ln P) / \ln r)$  in the average case, where  $P$  is the population size and  $r$  is the fitness ratio. Elhadef and Ayeb also proposed in Elhadef and Ayeb [2002, 2001b] other comparison-based diagnosis algorithms, including a serial genetic algorithm. Abrougui and Elhadef in [2005] present a parallel version of the existing evolutionary diagnosis models, and also present a parallel genetic diagnosis algorithm.



### 16.1. Artificial-Immune-System Comparison-Based Diagnosis

An Artificial Immune System (AIS) is designed to mimic the operations of the human immune system which protects the body from the attacks of foreign organisms such as bacteria and viruses. The design of an AIS is quite similar to the design of other traditional computational intelligence approaches, such as genetic algorithms. These systems have been used in many applications, including classical system-level diagnosis [Amaral et al. 2004; Dasgupta et al. 2004; Ishida 1997].

Elhadef et al. [2006c] argue that the genetic diagnosis algorithm suffers from a loss in population diversity due especially to the use of an adaptive mutation operator. This characteristic causes a very large worst-case running time compared to the average case. The authors solve this problem under the comparison-based diagnosis model by presenting an artificial-immune-system-based approach which preserves the population diversity, avoiding the worst case of the GA-based algorithm.

## 17. COMPARISON-BASED DIAGNOSIS APPLIED TO AD HOC NETWORKS

Mobile ad hoc networks (MANETs) implement a distributed cooperative environment consisting of different wireless and mobile devices (nodes), ranging from little hand-held devices to laptops. These networks are based on a peer-to-peer paradigm. Given the limited range of wireless communication, the network is generally multihop, since direct communication between nodes is generally not available. Communications are based on the one-to-many paradigm, that is, when a node transmits, all nodes in its transmission range receive the message. A major problem arises from the mobility of nodes causing the network topology to be variable and to some extent unpredictable. In fact, communication links between nodes may be broken, nodes may fail and possibly recover from failures, and new links may appear [Basile et al. 2003; Hollick et al. 2004; Albini et al. 2006]. Furthermore, nodes rely on on-board batteries for energy supply and the effect of battery depletion is similar to a crash fault.

Comparison-based diagnosis has been applied for mobile ad hoc networks by Chessa and Santi in [2001] and Elhadef et al. in [2006a, 2007]. Protocols for both models are able to identify *hard* (permanent) and *soft* (temporary) faults. A node suffers a hard fault when it ceases completely to communicate with other nodes. On the other hand, a node affected by a soft fault continues to operate and communicate, but with altered behavior. The description of both models follows.

### 17.1. Chessa and Santi's [2001] Diagnostic Model

The model proposed by Chessa and Santi [2001] is based on the MM\* model. They present two implementations of the model. In the first, the network topology does not change during diagnosis. In the second, the network topology is allowed to change during diagnosis.

The topology of system at time  $\tau$  is modeled as a direct graph  $G(\tau) = (V, L(\tau))$ , where  $V$  is the set of nodes and  $L(\tau)$  is the set of logical links at time  $\tau$ . Given any two nodes  $u, v \in V$ , there is an edge ( $u \rightarrow v \in L(\tau)$ ) if and only if  $v$  is in the transmission range of  $u$  at time  $\tau$ . The model considers only bidirectional links, thus if ( $u \rightarrow v \in L(\tau)$ ) then ( $v \rightarrow u \in L(\tau)$ ), and  $G_\tau = (V, L(\tau))$  is undirected. The set of nodes inside the transmission range of a given node  $u$  at time  $\tau$  is called the neighbor set of  $u$  at time  $\tau$ , denoted by  $N(u, \tau)$  or simply  $N(u)$ .

This model makes the following assumptions.

- (1) Each node has a unique identifier;

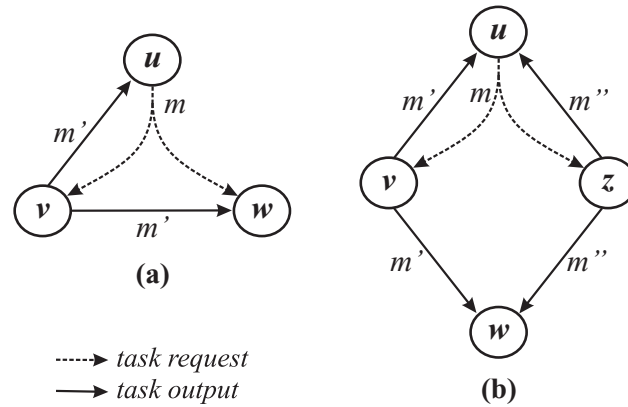


Fig. 29. (a) Unit  $w$  received a test request  $m$  from  $u$ ; (b) unit  $w$  received test responses  $m'$  and  $m''$  to test request  $m$ .

- (2) there is a link-level protocol that provides the following:
- solves contentions;
  - provides one-hop reliable broadcast, called  $1rb(\cdot)$ ;
  - identifies the sender of a received message.

Comparisons between units take advantage of the shared nature of the communication link. A fault-free unit  $u$  (the tester) sends test requests to its neighbors and waits for their responses. As the responses are received, units are diagnosed based on the comparison assumptions of the MM\* model.

Depending on the assumptions regarding the network topology, different decisions on the state (faulty or fault free) of the units that did not reply to the test request can be taken, as described next.

**17.1.1. Fixed Topology Comparison Protocol.** This protocol, also called Static Distributed Self-Diagnosis Protocol (Static-DSDP), assumes that the network topology does not change during test execution, that is, if unit  $u$  sends a test request at time  $\tau$ , and  $T_{out}$  is the timeout for this test, then  $N(u, \tau') = N(u, \tau) = N(u)$  for any  $\tau < \tau' \leq \tau + T_{out}$ . This assumption does not mean that the network is static, rather that its topology does not change during diagnosis: nodes are allowed to move, but they cannot migrate out of their neighbor's transmitting ranges.

Comparisons are performed based on the following protocol.

- Test request generation:* at time  $\tau$ , unit  $u$  generates a test sequence number  $i$ , a task  $T_i$ , the expected result  $R_{u,i}$ , and sends the message  $m = (u, i, T_i)$  to  $N(u, \tau)$  using the primitive  $1rb(m)$ .
- Test request reception:* any unit  $v \in N(u)$ , upon receiving  $m$ , generates the result  $R_{v,i}$  for  $T_i$  and invokes  $1rb(m')$  at time  $\tau'$ , with  $\tau < \tau' \leq \tau + T_{out}$ . Message  $m' = (u, i, R_{v,i})$  is the test response, and  $(u, i)$  is the header of the test response.
- Test response reception:* every unit  $w \in N(v)$  receives  $m'$ . As  $u \in N(v)$ ,  $u$  also receives  $m'$ , and compares  $R_{u,i}$  and  $R_{v,i}$ :  $v$  is diagnosed as fault free if the outcome is 0, and as faulty otherwise. For  $w \in N(v)$  but  $w \neq u$ , either  $w \in N(u)$  or  $w \notin N(u)$ . In case  $w \in N(u)$ , as shown in Figure 29(a),  $w$  compares  $R_{v,i}$  and  $R_{w,i}$ :  $v$  is diagnosed as fault free if the comparison outcome is 0, as faulty otherwise. In case  $w \notin N(u)$ , as shown in Figure 29(b), if  $w$  has also received another message  $m''$  about the same task from node  $z \in N(u)$ ,  $w$  compares  $R_{z,i}$  and  $R_{v,i}$ . If the comparison indicates a match, then nodes  $v$  and  $z$  are diagnosed as fault free. If the comparison results in a mismatch,

and  $z$  has been diagnosed as fault free, then  $v$  is diagnosed as faulty. Finally in case  $w \notin N(u)$  and  $w$  has not received another message about the same task, then  $R_{v,i}$  is just stored locally.

—*Timeout*: At time  $\tau + T_{out}$  node  $u$  diagnoses all other nodes that did not reply the test request as faulty.

Authors show that assuming a fixed network topology, if a fault-free node  $u$  generates a test request at time  $\tau$ , then at time  $\tau + T_{out}$ :

- Node  $u$  has correctly diagnosed the state of all nodes in  $N(u)$ .
- Any fault-free unit  $v \in N(u)$  has correctly diagnosed the state of fault-free and “soft-faulted” units in  $N(u) \cap N(v)$ .
- Any fault-free unit  $z \in N_2(u)$ , where  $N_2(u) = \{z \in V - N(u) \text{ such that } |N(u) \cap N(z)| \geq 2\}$ , has correctly diagnosed the state of the fault-free and “soft-faulted” units in  $N(u) \cap N(z)$  if at least two units in  $N(u) \cap N(z)$  are fault free.

*17.1.2. Time-Varying Topology Comparison Protocol.* Assume now that nodes are allowed to migrate during test execution. Comparisons are performed according to the following dynamic, that is, time varying, topology protocol.

- Test request generation*: at time  $\tau$ , unit  $u$  (tester) generates a test sequence number  $i$ , a task  $T_i$ , the expected result  $R_{u,i}$ , and sends message  $m = (u, i, T_i)$  to  $N(u, \tau)$  using  $1rb(m)$ .
- Test request reception*: any unit  $v \in N(u, \tau)$ , upon receiving  $m$ , generates the result  $R_{v,i}$  for  $T_i$  and invokes  $1rb(m')$  at time  $\tau'$ , with  $\tau < \tau' \leq \tau + T_{out}$ , where  $m' = (u, i, R_{v,i})$ .
- Test response reception*: any unit  $w \in N(v, \tau)$ , upon receiving  $m'$ , does the following: if  $w = u$ , it compares  $R_{v,i}$  with the expected result  $R_{u,i}$  and generates the comparison outcome. Unit  $v$  is diagnosed as fault free if the outcome is 0, and as faulty otherwise. If  $w \neq u$ , the following cases arise: (a)  $w \in N(u, \tau)$ . In this case, unit  $w$  received the test request  $m$  from  $u$ , hence it can compare  $R_{v,i}$  with  $R_{w,i}$ . Unit  $v$  is diagnosed as fault free if the comparison outcome is 0, and as faulty otherwise. (b)  $w \notin N(u, \tau)$ . Unit  $v$  is not “hard-faulted”, and its test response is compared to test responses received for the same task, if one exists. If there is some  $z \in N(u)$  such that  $R_{z,i} = R_{v,i}$  then both nodes are diagnosed as fault free; otherwise, if  $z$  has been diagnosed as fault free, then  $v$  is diagnosed as faulty. Otherwise, the test result  $R_{v,i}$  is stored.
- Timeout*: At time  $\tau + T_{out}$  node  $u$  diagnoses all other nodes that did not reply the test request as faulty.

Since the topology of the network varies with time, in general  $N(u, \tau) \neq N(u, \tau + T_{out})$ . As a consequence, “hard-faulted” units cannot be distinguished from fault-free units, that migrated out of the testing units’ transmitting range. For this reason, the tester can only classify the units that did not reply to its test request.

The authors show that if fault-free node  $u$  generates a test request at time  $\tau$ , and the network topology can change during diagnosis, then, at time  $\tau + T_{out}$ , node  $u$  has correctly diagnosed the state of all fault-free or “soft-faulted” nodes in  $N(u, \tau) \cap N(u, \tau + T_{out})$ .

## 17.2. Elhadeif et al. [2007, 2006a] Diagnostic Model

In Elhadeif et al. [2007, 2006a] the authors present comparison-based diagnosis protocols for mobile ad hoc networks. Two protocols are presented: the Adaptive Distributed Self-Diagnosis Protocol (Adaptive-DSDP) for fixed topology networks, and the Mobile Distributed Self-Diagnosis Protocol (Mobile-DSDP) for time-varying topology networks.

The key idea of both protocols is that a node, when replying to a test request, should also provide the test task along with its output for that test. Thus any receiver would be able to diagnose its state by simply comparing this output to similar outputs for the same test, or even by comparing the received result to its own output after performing the same test.

Besides the fact that nodes forward tasks with test responses, the fixed-topology diagnosis model on which Adaptive-DSDP is based also differs from Chessa and Santi's model [2001] in their dissemination strategies. In Chessa and Santi's [2001] model, once a node collects all its neighbors responses, it forwards its local view to all other nodes in the MANET using a flooding-based dissemination phase. On the other hand, Adaptive-DSDP uses a spanning tree and a gossip-style dissemination strategy [Elhadef et al. 2007].

The new time-varying topology comparison protocol is described next. In this protocol, a node's neighbors are classified as either stable or dynamic. Dynamic neighbors are those that have just moved to the node's neighborhood.

- Test request generation:* Node  $u$  transmits test requests to its neighbors at a given time  $\tau$ . The test request includes a task,  $T_i$ , where  $i$  is a sequence number. After sending the test request,  $\langle Test, T_u \rangle$ , a timer is set to  $T_{out}$ . In addition, a second timer is set to  $T_{DiagnosisSession}$ , which refers to the worst-case diagnosis latency if all nodes are fault free. This second timer is used to identify "hard-faulted" dynamic nodes that did not reply to test requests or that may have moved enough so that their states was not diagnosed by other nodes.
- Test request reception:* When node  $v$  receives a test request from one of its neighbors,  $u$ , it behaves in the following way. If it already knows the outcome  $R$  of the test task  $T_u$ , then it sets  $R_u^v = R$ ; otherwise, it performs the task  $T_u$  and generates its result  $R_u^v$ . Then, it transmits to all its neighbors the test response message  $\langle Response, T_u, R_u^v \rangle$ , which contains the test task  $T_u$  and the result  $R_u^v$ . The test response is stored in a response set, denoted by  $Validated_v$ , in which all correct test responses, either generated by the node itself or deduced during the diagnosis session, are maintained. At this stage, node  $v$  generates its own test request, if not done yet, and sends it to all its neighbors. Each node is required to reply to at most  $t + 1$  test requests, if the system is  $t$ -diagnosable.
- Test response reception:* When dealing with test responses, different scenarios should be considered. In fact node  $w$  may receive test responses from its stable and dynamic neighbors. Whether they are stable or dynamic neighbors, the fact that  $w$  has received the tasks outputs provided by these neighbors along with the test tasks that have been computed to generate these results, node  $w$  will be able to diagnose their status. All test responses received by  $w$  for which it is unable to classify as correct are stored in a pending set, called  $Pending_w$ .
- Timeout:* Upon the occurrence of the first timeout,  $T_{out}$ , node  $u$  is able to diagnose the status of its stable neighbors as well as that of dynamic neighbors from which it has already received at least one test response. At this stage, node  $u$  disseminates its local diagnostic view to all its neighbors. When the second timeout occurs,  $T_{DiagnosisSession}$ , node  $u$  will consider all remaining nodes to be faulty.

Elhadef et al. present in [2006b] another distributed comparison-based self-diagnosis protocol for wireless ad hoc networks based on Chessa and Santi's [2001] model. The proposed protocol is called Dynamic-DSDP which also identifies hard and soft faults.

They compare the Dynamic-DSDP protocol with Chessa and Santi's [2001] Static-DSDP protocol. Consider the following three definitions. (1)  $T_{gen}$  is the elapsed time upper bound between the reception of the first diagnostic message and the generation

of the corresponding test request. (2) A diagnostic message can be a test request, a test response, a timeout message, or a dissemination message. (3)  $T_f$  is an upper bound to the time needed to propagate a dissemination message. The Dynamic-DSDP protocol has time complexity  $O(\Lambda(T_{gen} + d_{ST}T_f) + T_{out})$  while the Static-DSDP protocol is  $O(\Lambda(T_{gen} + T_f) + T_{out})$ , where  $\Lambda$  denotes the diameter of graph  $G$  and  $d_{ST}$  is the depth of the spanning tree used to disseminate messages. Furthermore, Dynamic-DSDP has communication complexity  $O(Nk_G) \simeq O(Nt)$  while Static-DSDP requires  $O(N(N + 1 + \Delta)) \simeq O(N^2)$  messages, where  $\Delta$  is the maximum node degree and  $k_G$  denote the connectivity of  $G$ .

The authors also prove the complexity of Mobile-DSDP in Elhadeef et al. [2006a] and present the analysis of Adaptive-DSDP in Elhadeef et al. [2007]. Mobile-DSDP has time complexity  $O(\hat{\Delta}(T_{gen} + T_f) + T_{out})$  and requires  $O(N(N + \hat{k}))$  messages, where  $\hat{\Delta}$  and  $\hat{k}$  denote, respectively, the maximum diameter and the minimum connectivity of graph  $G$ . Adaptive-DSDP has time complexity  $O(\Lambda T_{gen} + (d_{ST} + N - 1)T_f + T_{out})$  and has communication complexity  $O(N\Delta)$ .

## 18. A SUMMARY OF COMPARISON-BASED DIAGNOSIS RESULTS

This section summarizes relevant comparison-based diagnosis results presented in this survey. The three graphs in Figure 30, Figure 31, and Figure 32 show the relationship among the several results. A vertex of any of these graphs represents either a model, an algorithm, or other relevant result. Each vertex has two labels, the inside label listing the authors and the outside label listing contributions of the respective work. A directed edge from vertex  $a$  to vertex  $b$  denotes that the result identified by vertex  $b$  is based on, is an extension of, or is related to vertex  $a$ 's result. Furthermore, the graphs in those figures are ordered in a chronological fashion. The three graphs are related to each other.

Four vertices appear in all three graphs, namely Malek [1980], Chwa and Hakimi [1981b], Maeng and Malek [1981], and Sengupta and Dahbura [1992], which are drawn with dots. These four vertices represent intersections of the timelines in the three figures. The first Figure 30 shows the results derived from early comparison-based models: both Malek's [1980] and Chwa and Hakimi's [1981b] models. Figure 31 shows the results based on the MM model; and, subsequently Figure 32 shows results based on the MM\* model.

Table V contains a more detailed summary of all comparison-based results. All results are grouped by the model they are based on. The table has three columns. The first column indicates the comparison-based model. The next two columns give, respectively, the reference and contributions.

## CONCLUSIONS

This work presented a comprehensive and integrated view of comparison-based diagnosis results including models, algorithms, diagnosability bounds, and applications. These results have been published for a period that spans the past three decades. It is important to highlight that both theoretical results and promising new applications have been proposed recently. A uniform notation for describing all these results was employed. In comparison-based system-level diagnosis tasks are assigned to and executed by pairs of units. The task outputs are returned and then compared. Depending on the comparison outcomes, units are classified as faulty or fault free.

This survey described how the several models for comparison-based diagnosis differ, that is, in terms of assumptions, on how tasks are assigned, how outcomes are returned, where task outputs are compared, and how results are interpreted. Models

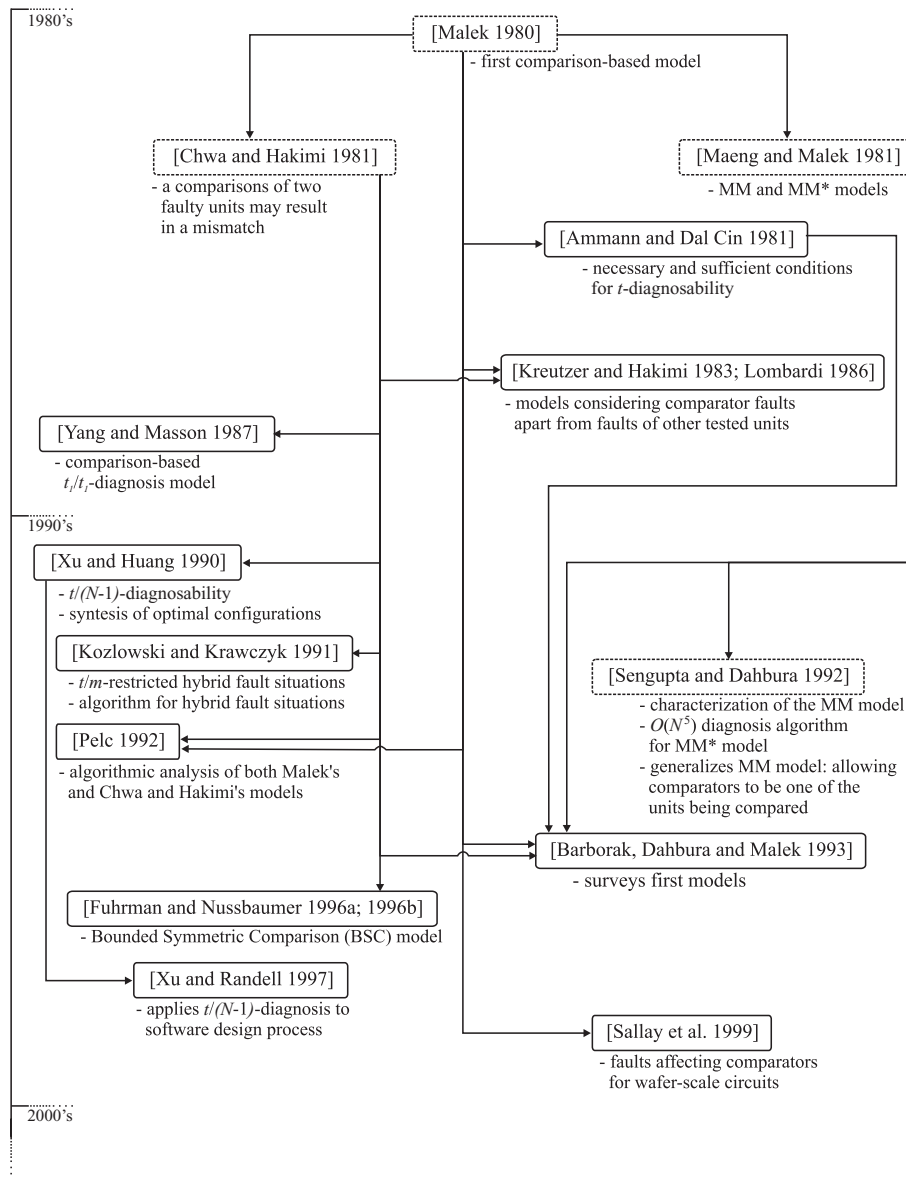


Fig. 30. Comparison-based diagnosis timeline: results based on early models.

either assume that only the task execution is distributed, or, alternatively, that also comparisons and the diagnosis itself are distributed among the system units. Some models work under probabilistic assumptions. The diagnosability of several popular interconnection network topologies under comparison-based models was also presented. The objective was to describe not only models but also algorithms in a way to help readers to understand each contribution and how it relates to the field as a whole. A range of applications have been described, including the detection of unauthorized modifications for replicated data, determining faulty processors in parallel

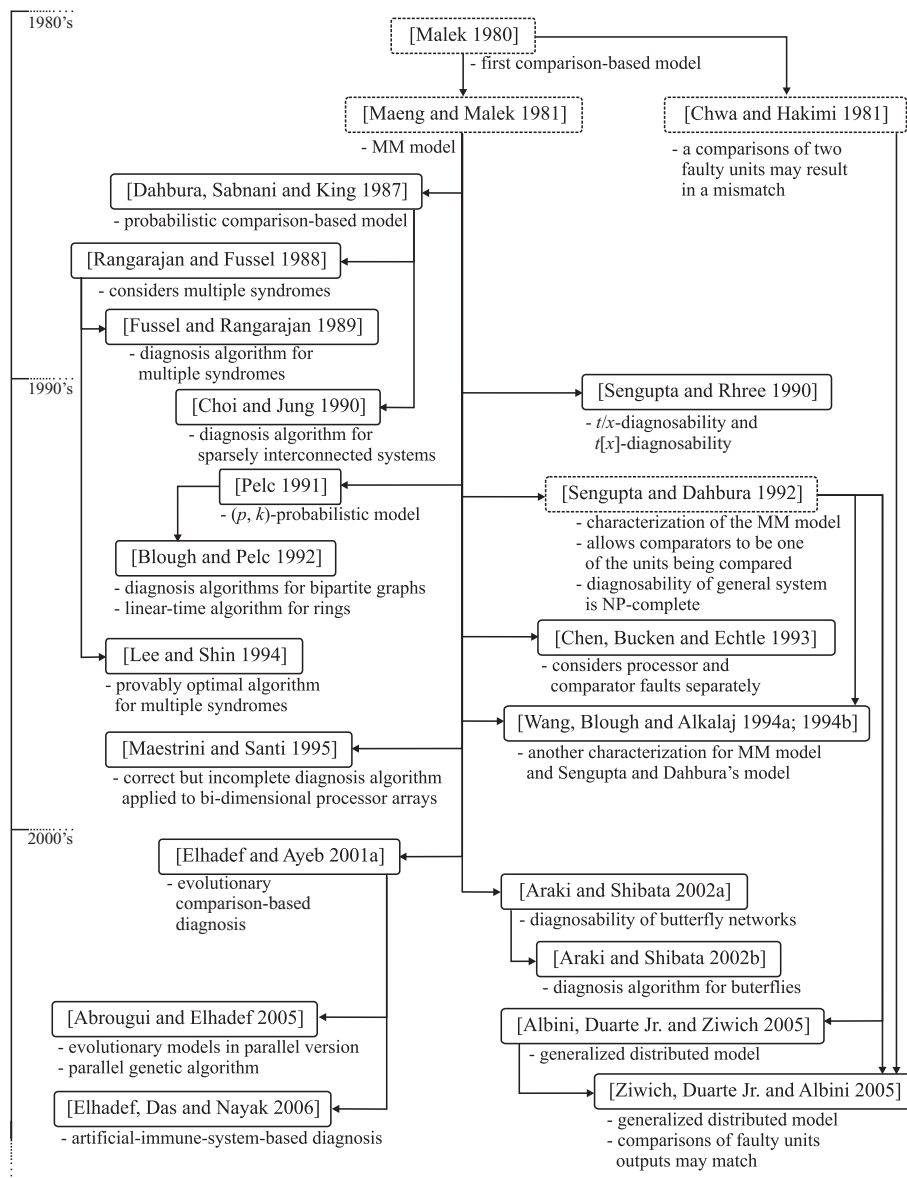


Fig. 31. Comparison-based diagnosis timeline: results based on the MM model.

architectures, monitoring task outcomes in grid systems, and the diagnosis of mobile ad hoc networks.

Besides integrating and clarifying comparison-based diagnosis results, the main objective of the survey is to ignite the potential of these models, methods, and technology, which we believe can bring novel contributions to diverse fields. In security for instance, comparison-based diagnosis can be used for checking the integrity of data and services; in software engineering, a framework for comparisons can be employed in several variations of software testing [Pressman 2004], such as mutation, perturbation,

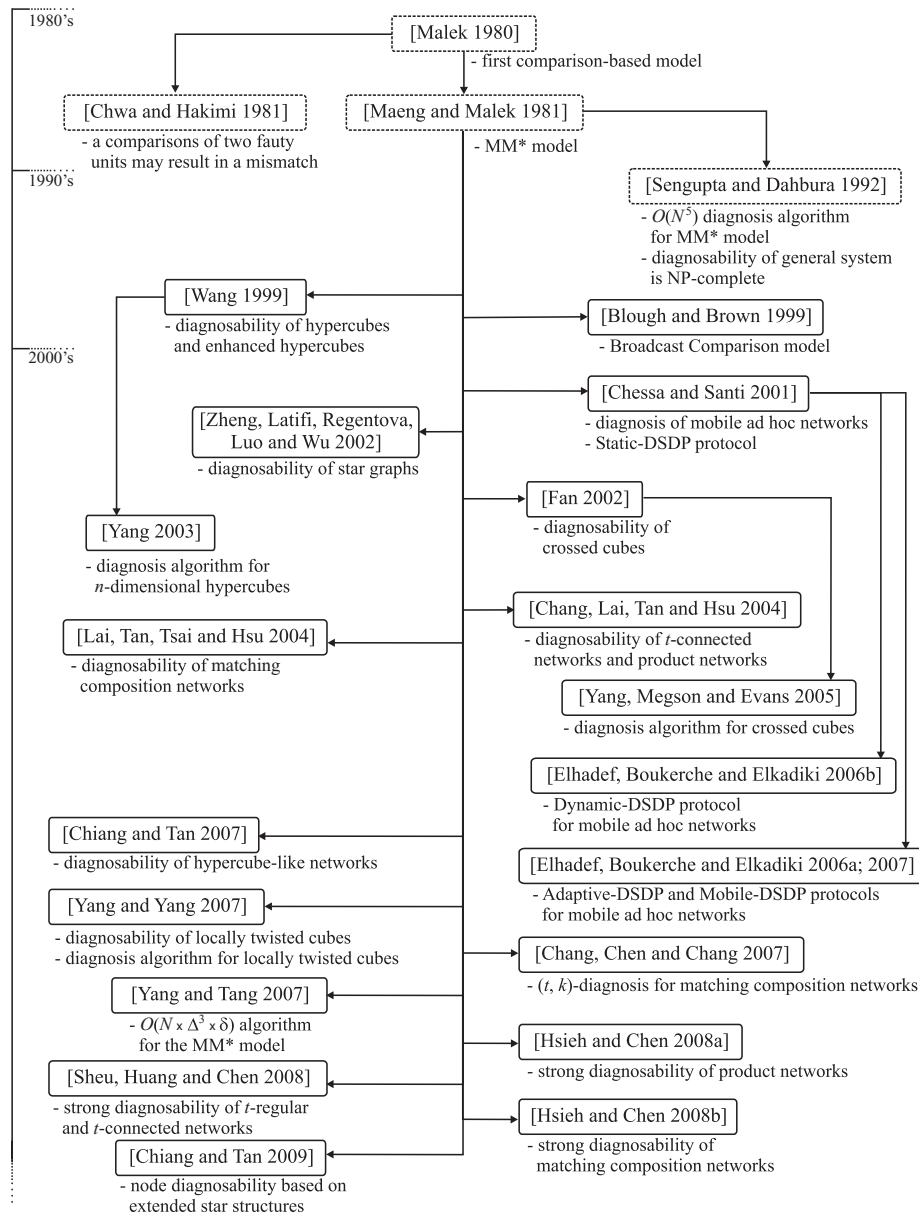


Fig. 32. Comparison-based diagnosis timeline: results based on the MM\* model.

and regression testing. Furthermore, in multicore and cloud computing systems, innovative ways to diagnosis and fault tolerance can be pursued.

#### ACKNOWLEDGMENTS

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Table V. Summary of Comparison-Based Results

Model	Reference	Main Contributions
Malek's model	[Malek 1980]	<ul style="list-style-type: none"> <li>- first comparison-based model</li> <li>- compared units are different</li> <li>- the comparison of one or two faulty units results in a mismatch</li> <li>- central observer is a trusted unit that executes comparisons and performs the diagnosis</li> <li>- the diagnosability is <math>N - 2</math></li> </ul>
	[Ammann and Dal Cin 1981]	<ul style="list-style-type: none"> <li>- necessary and sufficient conditions for <math>t</math>-diagnosability</li> </ul>
	[Sallay et al. 1999]	<ul style="list-style-type: none"> <li>- strategy to identify faults affecting comparators</li> <li>- application for wafer-scale circuits</li> </ul>
	[Pelc 1992]	<ul style="list-style-type: none"> <li>- algorithmic analysis of both Malek's and Chwa and Hakimi's models</li> <li>- worst case number of tests for optimal algorithms for <math>t</math>-diagnosis, sequential <math>t</math>-diagnosis and one-step <math>t</math>-diagnosis for both models, under non-adaptive and adaptive testing</li> </ul>
	[Barborak et al. 1993]	<ul style="list-style-type: none"> <li>- surveys early models</li> </ul>
Chwa and Hakimi's model	[Chwa and Hakimi 1981b]	<ul style="list-style-type: none"> <li>- the comparison of two faulty units may result in a match</li> </ul>
	[Fuhrman and Nussbaumer 1996b, 1996a]	<ul style="list-style-type: none"> <li>- Bounded Symmetric Comparison model, considers a limit on the number of faulty units that can produce identical results</li> </ul>
	[Kozlowski and Krawczyk 1991]	<ul style="list-style-type: none"> <li>- extension of Chwa and Hakimi's model for <math>t/m</math>-restricted hybrid fault situations</li> </ul>
	[Yang and Masson 1987]	<ul style="list-style-type: none"> <li>- comparison-based <math>t_1/t_1</math>-diagnosis model</li> </ul>
	[Xu and Huang 1990]	<ul style="list-style-type: none"> <li>- characterization of <math>t/(N - 1)</math>-diagnosability under Chwa and Hakimi's model</li> <li>- synthesis of optimal <math>t/(N - 1)</math>-diagnosable configurations for topologies such as chains and loops</li> </ul>
	[Xu and Randell 1997]	<ul style="list-style-type: none"> <li>- application of <math>t/(N - 1)</math> diagnosis to the software design process</li> </ul>
	[Kreutzer and Hakimi 1983; Lombardi 1986]	<ul style="list-style-type: none"> <li>- models considering comparator faults apart from faults of other tested units</li> <li>- characterization of the proposed models, <math>(t - t_c)</math>-diagnosability</li> </ul>
MM model	[Maeng and Malek 1981]	<ul style="list-style-type: none"> <li>- comparison diagnosis model in which units are also comparators</li> <li>- comparison outputs when at least one unit is faulty always results in a mismatch</li> <li>- central observer is a trusted unit that performs diagnosis</li> <li>- necessary and sufficient conditions for one-step <math>t</math>-diagnosability</li> <li>- procedure to construct minimal graph for diagnosable systems</li> <li>- evaluation of diagnosis latency in terms of test cycles</li> </ul>
	[Sengupta and Dahbura 1992]	<ul style="list-style-type: none"> <li>- generalization of the MM model: allows comparators to be one of the units being compared</li> <li>- characterization of diagnosable systems under the MM model</li> <li>- diagnosability of general systems is NP-complete</li> </ul>

(Continued on next page)

Table V. (Continued)

Model	Reference	Main Contributions
MM Model (continued)	[Sengupta and Rhee 1990]	- $t/x$ -diagnosability and $t[x]$ -diagnosability
	[Chen et al. 1993]	- extension of MM model considering processor and comparator faults separately; diagnosability evaluation
	[Wang et al. 1994a, 1994b]	- new necessary and sufficient diagnosability conditions for both the MM model and Sengupta and Dahbura's model
	[Maestrini and Santi 1995]	- correct but incomplete diagnosis algorithm applied to locate faults in bi-dimensional processor arrays
	[Araki and Shibata 2002a]	- diagnosability of $k$ -ary $r$ -dimensional butterfly networks
	[Araki and Shibata 2002b]	- $O(k^2n)$ diagnosis algorithm for butterfly networks
MM* Model	[Maeng and Malek 1981]	- MM* model is a special case of the MM model: each unit compares all pairs of neighbors
	[Sengupta and Dahbura 1992]	- diagnosis algorithm with time complexity $O(N^5)$ under the MM* model - diagnosability of general systems under the MM* model is NP-complete
	[Yang and Tang 2007]	- diagnosis algorithm with time complexity $O(N \times \Delta^3 \times \delta)$ under the MM* model, where $\Delta$ and $\delta$ are respectively the maximum and the minimum degrees of a node
	[Wang 1999]	- diagnosability of hypercubes and enhanced hypercubes
	[Yang 2003]	- worst case $O(N \log_2^2 N)$ diagnosis algorithm for hypercubes
	[Fan 2002]	- diagnosability of crossed cubes
	[Yang et al. 2005b]	- $O(N \log_2^2 N)$ diagnosis algorithm for crossed cubes
	[Yang and Yang 2007]	- diagnosability of locally twisted cubes - $O(N \log_2^2 N)$ diagnosis algorithm for locally twisted cubes
	[Chiang and Tan 2007]	- diagnosability of hypercube-like networks
	[Zheng et al. 2002]	- diagnosability of star graphs
	[Lai et al. 2004]	- diagnosability of matching composition networks
	[Chang et al. 2007]	- $(t, k)$ -diagnosis for matching composition networks
	[Chang et al. 2004]	- diagnosability of $t$ -connected networks - diagnosability of product networks
	[Sheu et al. 2008]	- strong diagnosability of $t$ -regular and $t$ -connected networks
	[Hsieh and Chen 2008a]	- strong diagnosability of product networks: hypercubes, mesh-connected $k$ -ary $n$ -cubes, torus-connected $k$ -ary $n$ -cubes, hyper-Petersen networks
	[Hsieh and Chen 2008b]	- strong diagnosability of matching composition networks: $n$ -dimensional crossed cubes, Möbius cubes, twisted cubes and locally twisted cubes
[Chessa and Santi 2001]	- comparison-based diagnosis applied for mobile ad hoc networks	
[Elhadef et al. 2006b]	- Static-DSDP protocol for fixed topology - protocol Dynamic-DSDP for ad hoc networks based on Chessa and Santi's model	

Table V. (Continued)

Model	Reference	Main Contributions
MM* Model (continued)	[Elhadeh et al. 2006a, 2007]	- comparison-based diagnosis applied for mobile ad hoc networks - Adaptive-DSDP Protocol for fixed topology networks - Mobile-DSDP protocol for time-varying topology networks
	[Chiang and Tan 2009]	- node diagnosability based on extended star structures
Broadcast Comparison Model	[Blough and Brown 1999]	- fully distributed comparison model - based on MM* for systems with reliable broadcast - polynomial-time algorithms to diagnose static and dynamic fault situations
Generalized Distributed models	[Albini et al. 2005; Albini and Duarte Jr. 2001]	- the generalized distributed comparison-based model: a hierarchical, adaptive and distributed model based on Sengupta and Dahbura's model - <i>Hi-Comp</i> diagnosis algorithm: requires at most $O(N^3)$ comparisons and has worst-case latency of $O(\log_2 N)$ rounds
	[Ziwich et al. 2005]	- generalized distributed comparison-based model assuming the comparison of faulty units outputs may match - <i>Hi-Dif</i> diagnosis algorithm that requires at most $O(N^2)$ comparisons and has worst-case latency of $O(\log_2 N)$ latency
Probabilistic model	[Dahbura et al. 1987]	- probabilistic comparison based model - considers probabilities for a match or a mismatch when comparing units
	[Rangarajan and Fussell 1988]	- strategy based on the evaluation of multiple syndromes
	[Fussell and Rangarajan 1989]	- $O(\log_2 N)$ for the evaluation of multiple syndromes
	[Lee and Shin 1994]	- probably optimal algorithm for the evaluation of multiple syndromes
	[Choi and Jung 1990]	- diagnosis algorithm for sparsely interconnected systems
$(p, k)$ -Probabilistic model	[Pelc 1991]	- a task has $k$ possible outcomes - each unit has the same probability $p < 1/2$ - probability of obtaining a match when comparing a faulty unit and a fault-free unit or two faulty units is $q = 1/k$ - <i>diagnosis</i> and the <i>diagnosability</i> problems are NP-hard for general topology
	[Blough and Pelc 1992]	- polynomial time diagnosis algorithms for bipartite graphs (includes hypercubes, grids and forests) - linear-time algorithm to perform optimal diagnosis of rings
Evolutionary Comparison-Based models	[Elhadeh and Ayebe 2001a]	- evolutionary comparison-based diagnosis
	[Abrougui and Elhadeh 2005]	- parallel evolutionary diagnosis models
	[Elhadeh et al. 2006c]	- comparison-based diagnosis model with an artificial-immune-system-based approach

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