Recognition of Biclique Graphs of Some Graph Classes

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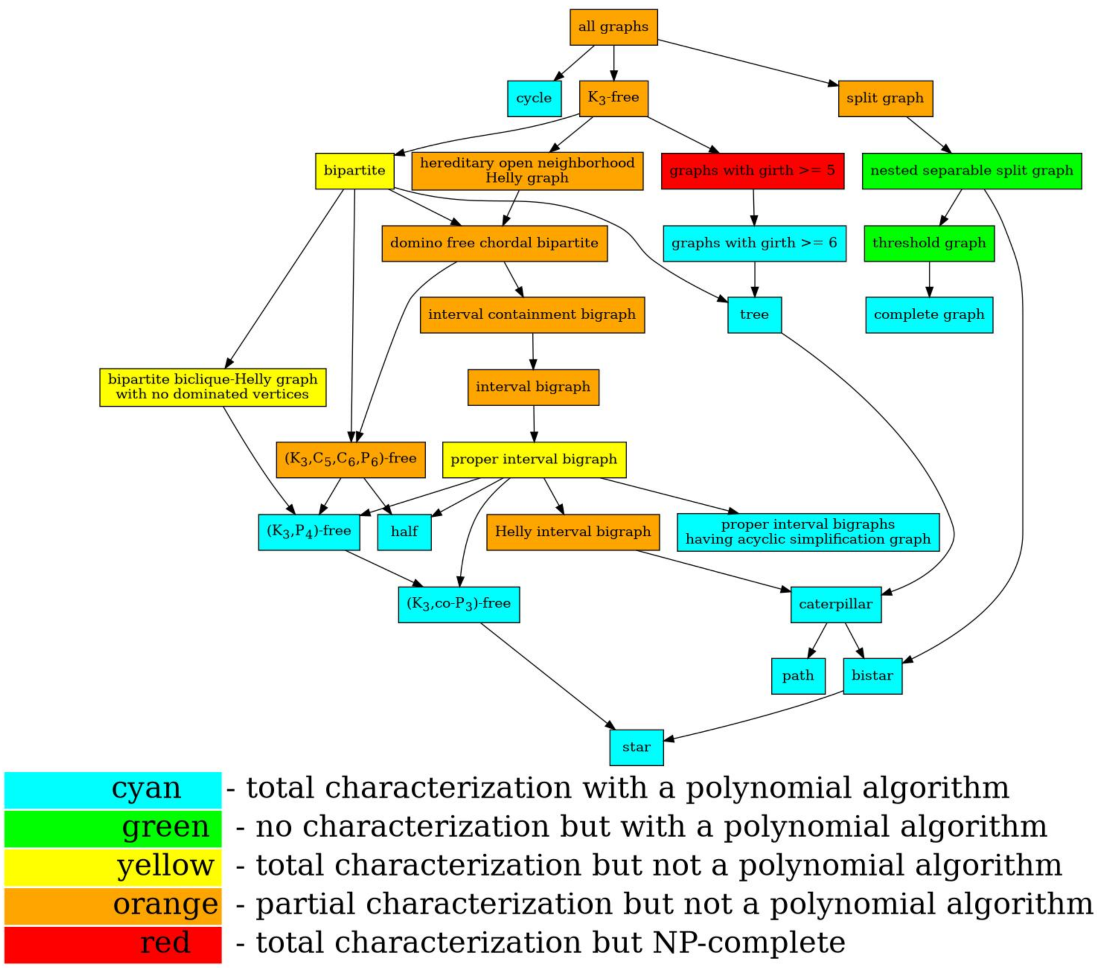

Figure 1: Graph classes and the status of the recognition of its biclique graph

Glossary

Classes

- *BBHGD*: Bipartite biclique-Helly graphs with no dominated vertices [13]

Table 1: Biclique graph of some classes. At column "$KB(G)$, $G \in \mathcal{A}$" we can find a brief description of the graph $KB(G)$ for each class; at column "class $KB(\mathcal{A})$" appears the class that is equal to (or a super-class of) $KB(\mathcal{A})$. At column "complexity" we present the complexity (if known) of recognizing $KB(\mathcal{A})$.

class $\mathcal{A}$	$KB(G)$**,** $G \in \mathcal{A}$	**class** $KB(\mathcal{A})$	**complexity**
complete	$L(G)$ [1]	L(complete) [1]	$\mathcal{P}$ [16, 18, 19]
tree	$(G - leaves(G))^2$ [1]	(tree)^2 [1]	$\mathcal{P}$ (linear) [17]
path (P_n)	$\emptyset$, for $n = 1$ K_1, for $n = 2$ $(P_{n-2})^2$, for $n > 2$ [1]	(path)^2 [1]	$\mathcal{P}$ (linear) [17]
half (with $2n$ vertices) [5]	K_n	complete	$\mathcal{P}$ (linear)
star	K_1	K_1	$\mathcal{P}$ (constant)
bistar	K_2	K_2	$\mathcal{P}$ (constant)
(K_3, P_4)-free graphs	$\mathcal{C}_{>1}(G)K_1$	nK_1 [4, 3]	$\mathcal{P}$ (constant)
$(K_3, \text{co-}P_3)$-free graphs	K_1	K_1	$\mathcal{P}$ (constant)
caterpilar	$(G - leaves(G))^2$	(path)^2	$\mathcal{P}$ (linear) [17]
cycle (C_n)	K_1, for $n = 4$ $(C_n)^2$, for $n \neq 4$ [1]	(cycle)$^2 - K_4 + K_1$ [1]	$\mathcal{P}$ [6]
$\mathcal{G}_k$, for $k \geq 5$	$(G - leaves(G))^2$ [1]	$(\mathcal{G}_k)^2$, for $k \geq 5$ [1]	$\mathcal{P}$, for $k \geq 6$ $\mathcal{NP}$-complete, for $k = 5$ [6]
ICB	**OPEN**	$\subset (IIC - PG)^2$ [4, 3, 2]	*
IBG	**OPEN**	$\subset (IIC - PG)^2$ [4, 3] $\subset K_{1,4}$-free co-CG [1]	**OPEN**
PIB	$(L(S(G))^2$ [1]	$(L(PIB))^2$ [1]	**OPEN**
PIB-ASG	$(L(S(G))^2$ [1]	1-PIG [1]	$\mathcal{P}$ [1]
HIB	$K(G^2)$ [13]	$\subset PIG \cap (L(PIB))^2$ [1, 9, 12, 14]	**OPEN**
(K_3, C_5, C_6, P_6)-free graphs	$K(G^2)$ [13]	—	**OPEN**
$DFCB$	$K(G^2)$ [13]	$\subset$ Dually Chordal	**OPEN**
$HONHG$	$K(G^2)$ [13]	—	**OPEN**
$BBHGD$	**OPEN**	$CHBDI$ [13]	**OPEN**
$NSSG$	—	—	$\mathcal{P}$ [7, 10]
threshold graphs	—	—	$\mathcal{P}$ [7, 10]
K_3-free graphs	$(KB_m(G))^2$ [8, 11]	$\subset \mathcal{G}^2$ [8, 11]	**OPEN**
bipartite	$(KB_m(G))^2$ [8, 11]	(IIC-comparability)^2 [8, 11] Characterization [13]	**OPEN**
$\mathcal{G}$	—	Characterization [13]	**OPEN**

- BPG: Bipartite permutation graphs [20] (= PIB [15])
- $CHBDI$: Clique independent Helly-bicovered with no dominated vertices graphs [13]
- co-CG: Co-comparability graphs
- C_n: Cycle with n vertices
- $DFCB$: Domino-free Chordal Bipartite [13]
- $\mathcal{G}$: All graphs
- $\mathcal{G}_k$: Graphs with girth at least k
- HIB: Helly interval bigraphs [9]
- $HONHG$: Hereditary open neighborhood Helly graphs (= (K_3, C_6)-free graphs) [13]
- IBG: Interval bigraphs [15]
- ICB: Interval containment bigraphs
- IIC-comparability: Interval intersection closed comparability graphs [8, 11]
- K_n: Complete graph of order n
- $NSSG$: Nested separable split graphs [7, 10]
- PG: Permutation Graphs
- PIB: Proper interval bigraphs (= BPG [15])
- PIB-ASG: Proper interval bigraphs having acyclic simplification graph [1]
- PIG: Proper interval graphs
- 1-PIG: 1-Proper interval graphs [1]
- P_n: Path with n vertices

Operators and Functions

- $\mathcal{C}_{>1}(G)$: Number of non-trivial components of G
- $K(G)$: Clique graph of G
- $L(G)$: Line graph of graph G
- $leaves(G)$: Set of leaves (vertices of degree 1) of graph G
- $S(G)$: Simplification graph of G [1]

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