

Matemática Discreta

Unidade 20: Recorrências (3)

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Generalização da ideia das unidades anteriores

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