Matemática Discreta

Unidade 21: Recorrências (4)

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Departamento de Informática da UFPR

Segundo Período Especial de 2020

Exercicio 101

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$$f(n) = \begin{cases} 1, & \text{se } n = 1, \\ f\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + 1, & \text{para todo } n \geq 2 \end{cases}$$

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$$\begin{aligned} h(n) &= \left\lfloor \frac{n}{2} \right\rfloor \\ s(n) &= 1 \\ n_0 &= 2 \\ h^k(n) &\overset{\text{C. 27}}{=} \left\lfloor \frac{n}{2^k} \right\rfloor \end{aligned}$$

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$$\begin{aligned} f(n) &= f(h^u(n)) + \sum_{i=0}^{u-1} s(h^i(n)) \\ &= f(h^u(n)) + \sum_{i=0}^{u-1} 1 \end{aligned}$$

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$$\left\lfloor \frac{n}{2^k} \right\rfloor < 2 \Leftrightarrow \left\lfloor \frac{n}{2^k} \right\rfloor \leq 1$$

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então

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