Matemática Discreta

Unidade 22: Recorrências (5)

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Departamento de Informática da UFPR

Segundo Período Especial de 2020

Exercício 102

$$f(n) = \begin{cases} 0, & \text{se } n = 0, \\ f\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + (n \bmod 2) & \text{se } n \geq 1 \end{cases}$$

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$$\begin{aligned} h(n) &= \left\lfloor \frac{n}{2} \right\rfloor \\ s(n) &= n \bmod 2 \\ n_0 &= 1 \\ h^k(n) &\overset{\text{C. 27}}{=} \left\lfloor \frac{n}{2^k} \right\rfloor \end{aligned}$$

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