Matemática Discreta

Unidade 25: Recorrências (8)

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Generalização das ideias das unidades anteriores

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\begin{aligned}
f(n) &= m(n)f(h(n)) + s(n) \\
&= m(n)(m(h(n))f(h^2(n) + s(h(n))) + s(n)
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&= \ldots
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&= \dots \\
&= f(h^u(n)) \prod_{i=0}^{u-1} m(h^i(n)) + \sum_{i=0}^{u-1} \left(s(h^i(n)) \prod_{j=0}^{i-1} m(h^j(n)) \right),
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\end{aligned}
$$

$u = \min \left\{ k \in \mathbb{N} \mid h^k(n) < n_0 \right\}$

Teorema 29

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