Matemática Discreta

Unidade 26: Recorrências (9)

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Exercicio 114

$$T^{-}(n) = \begin{cases} 0, & \text{se } n < 2, \\ 2\,T^{-}\left(\lfloor \frac{n}{2} \rfloor\right) + n - 1, & \text{se } n \geq 2 \end{cases}$$

Exercicio 114: $2T^{-}\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + n - 1,$ se $n \geq 2$

Exercicio 114: $2T^-\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + n - 1$, se $n \geq 2$

T. 29

$$T^-(n) = T^-(h^u(n)) \prod_{i=0}^{u-1} m(h^i(n)) + \sum_{i=0}^{u-1} s(h^i(n)) \prod_{j=0}^{i-1} m(h^j(n)), \text{ para todo } n \geq n_0,$$

Exercicio 114: $2T^-\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + n - 1$, se $n \geq 2$

T. 29

$$T^-(n) = T^-(h^u(n)) \prod_{i=0}^{u-1} m(h^i(n)) + \sum_{i=0}^{u-1} s(h^i(n)) \prod_{j=0}^{i-1} m(h^j(n)), \text{ para todo } n \geq n_0,$$

$$h(n) \quad = \quad \left\lfloor \frac{n}{2} \right\rfloor$$

Exercicio 114: $2T^-\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + n - 1$, se $n \geq 2$

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$$\begin{aligned} h(n) &= \left\lfloor \frac{n}{2} \right\rfloor \\ h^k(n) &= \left\lfloor \frac{n}{2^k} \right\rfloor \end{aligned}$$

Exercicio 114: $2T^-\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + n - 1$, se $n \geq 2$

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$$\begin{aligned} h(n) &= \left\lfloor \frac{n}{2} \right\rfloor \\ h^k(n) &= \left\lfloor \frac{n}{2^k} \right\rfloor \\ m(n) &= 2 \end{aligned}$$

Exercicio 114: $2T^-\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + n - 1$, se $n \geq 2$

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$$\begin{aligned} h(n) &= \left\lfloor \frac{n}{2} \right\rfloor \\ h^k(n) &= \left\lfloor \frac{n}{2^k} \right\rfloor \\ m(n) &= 2 \\ s(n) &= n - 1 \end{aligned}$$

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$$T^-(n) = T^-(h^u(n)) \prod_{i=0}^{u-1} m(h^i(n)) + \sum_{i=0}^{u-1} s(h^i(n)) \prod_{j=0}^{i-1} m(h^j(n)), \text{ para todo } n \geq n_0,$$

$$\begin{aligned} h(n) &= \left\lfloor \frac{n}{2} \right\rfloor \\ h^k(n) &= \left\lfloor \frac{n}{2^k} \right\rfloor \\ m(n) &= 2 \\ s(n) &= n - 1 \\ n_0 &= 2 \\ u &= \min\left\{ k \in \mathbb{N} \mid h^k(n) < n_0 \right\} \end{aligned}$$

Exercicio 114

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$h(n) = \lfloor \frac{n}{2} \rfloor$ $h^k(n) = \lfloor \frac{n}{2^k} \rfloor$ $m(n) = 2$ $s(n) = n - 1$ $n_0 = 2$

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$$T^-(n) = T^-(h^u(n)) \prod_{i=0}^{u-1} m(h^i(n)) + \sum_{i=0}^{u-1} s(h^i(n)) \prod_{j=0}^{i-1} m(h^j(n))$$

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$h(n) = \lfloor \frac{n}{2} \rfloor$ $h^k(n) = \lfloor \frac{n}{2^k} \rfloor$ $m(n) = 2$ $s(n) = n - 1$ $n_0 = 2$

$$T^-(n) = T^-(h^u(n)) \prod_{i=0}^{u-1} m(h^i(n)) + \sum_{i=0}^{u-1} s(h^i(n)) \prod_{j=0}^{i-1} m(h^j(n))$$

$$= T^- \left(\left\lfloor \frac{n}{2^u} \right\rfloor \right) \prod_{i=0}^{u-1} 2 + \sum_{i=0}^{u-1} s \left(\left\lfloor \frac{n}{2^i} \right\rfloor \right) \prod_{j=0}^{i-1} 2$$

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$$
\begin{aligned}
T^-(n) &= T^-(h^u(n)) \prod_{i=0}^{u-1} m(h^i(n)) + \sum_{i=0}^{u-1} s(h^i(n)) \prod_{j=0}^{i-1} m(h^j(n)) \\
&= T^- \left(\left\lfloor \frac{n}{2^u} \right\rfloor \right) \prod_{i=0}^{u-1} 2 + \sum_{i=0}^{u-1} s \left(\left\lfloor \frac{n}{2^i} \right\rfloor \right) \prod_{j=0}^{i-1} 2 \\
&= T^- \left(\left\lfloor \frac{n}{2^u} \right\rfloor \right) 2^u + \sum_{i=0}^{u-1} \left(\left\lfloor \frac{n}{2^i} \right\rfloor - 1 \right) 2^i
\end{aligned}
$$

Exercicio 114

$$T^{-}(n) = T^{-}\left(\left\lfloor \frac{n}{2^{u}} \right\rfloor\right) 2^{u} + \sum_{i=0}^{u-1} \left(\left\lfloor \frac{n}{2^{i}} \right\rfloor - 1\right) 2^{i}$$

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$$
\begin{aligned}
T^-(n) &= T^-\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) 2^u + \sum_{i=0}^{u-1} \left(\left\lfloor \frac{n}{2^i} \right\rfloor - 1\right) 2^i \\
&= 2^u T^-\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) + \sum_{i=0}^{u-1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - \sum_{i=0}^{u-1} 2^i
\end{aligned}
$$

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$$
\begin{aligned}
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&= 2^u T^-\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) + \sum_{i=0}^{u-1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - \sum_{i=0}^{u-1} 2^i \\
&= 2^u T^-\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) + \sum_{i=0}^{u-1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - (2^u - 1)
\end{aligned}
$$

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$$
\begin{aligned}
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&= 2^u T^-\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) + \sum_{i=0}^{u-1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - \sum_{i=0}^{u-1} 2^i \\
&= 2^u T^-\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) + \sum_{i=0}^{u-1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - (2^u - 1) \\
&= 2^u T^-\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) + \sum_{i=0}^{u-1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - 2^u + 1
\end{aligned}
$$

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$$u = \min \left\{ k \in \mathbb{N} \mid \left\lfloor \frac{n}{2^k} \right\rfloor < 2 \right\}$$

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$$T^-(n) = 2^u T^- \left(\left\lfloor \frac{n}{2^u} \right\rfloor \right) + \sum_{i=0}^{u-1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - 2^u + 1$$

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$$u = \min \left\{ k \in \mathbb{N} \mid \left\lfloor \frac{n}{2^k} \right\rfloor < 2 \right\} = \ldots = \lfloor \lg n \rfloor$$

$$\begin{aligned} T^-(n) &= 2^u T^- \left(\left\lfloor \frac{n}{2^u} \right\rfloor \right) + \sum_{i=0}^{u-1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - 2^u + 1 \\ &= 2^{\lfloor \lg n \rfloor} T^- \left(\left\lfloor \frac{n}{2^{\lfloor \lg n \rfloor}} \right\rfloor \right) + \sum_{i=0}^{\lfloor \lg n \rfloor - 1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - 2^{\lfloor \lg n \rfloor} + 1 \end{aligned}$$

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$$u = \min \left\{ k \in \mathbb{N} \mid \left\lfloor \frac{n}{2^k} \right\rfloor < 2 \right\} = \ldots = \lfloor \lg n \rfloor$$

$$\begin{aligned} T^-(n) &= 2^u T^- \left(\left\lfloor \frac{n}{2^u} \right\rfloor \right) + \sum_{i=0}^{u-1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - 2^u + 1 \\ &= 2^{\lfloor \lg n \rfloor} T^- \left(\left\lfloor \frac{n}{2^{\lfloor \lg n \rfloor}} \right\rfloor \right) + \sum_{i=0}^{\lfloor \lg n \rfloor - 1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - 2^{\lfloor \lg n \rfloor} + 1 \\ &= 2^{\lfloor \lg n \rfloor} T^-(1) + \sum_{i=0}^{\lfloor \lg n \rfloor - 1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - 2^{\lfloor \lg n \rfloor} + 1 \end{aligned}$$

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$$u = \min \left\{ k \in \mathbb{N} \mid \left\lfloor \frac{n}{2^k} \right\rfloor < 2 \right\} = \ldots = \lfloor \lg n \rfloor$$

$$\begin{aligned}
T^-(n) &= 2^u T^-\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) + \sum_{i=0}^{u-1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - 2^u + 1 \\
&= 2^{\lfloor \lg n \rfloor} T^-\left(\left\lfloor \frac{n}{2^{\lfloor \lg n \rfloor}} \right\rfloor\right) + \sum_{i=0}^{\lfloor \lg n \rfloor - 1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - 2^{\lfloor \lg n \rfloor} + 1 \\
&= 2^{\lfloor \lg n \rfloor} T^-(1) + \sum_{i=0}^{\lfloor \lg n \rfloor - 1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - 2^{\lfloor \lg n \rfloor} + 1 \\
&= \sum_{i=0}^{\lfloor \lg n \rfloor - 1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - 2^{\lfloor \lg n \rfloor} + 1
\end{aligned}$$