

Matemática Discreta

Unidade 22: Recorrências (5)

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Departamento de Informática da UFPR

Segundo Período Especial de 2020

Exercício 101

$$f(n) = \begin{cases} 0, & \text{se } n = 0, \\ f(\lfloor \frac{n}{2} \rfloor) + (n \bmod 2) & \text{se } n \geq 1 \end{cases}$$

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