

Matemática Discreta

Unidade 25: Recorrências (8)

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Generalização das ideias das unidades anteriores

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$$u = \min \{k \in \mathbb{N} \mid h^k(n) < n_0\}$$

Teorema 29

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