

Matemática Discreta

Unidade 26: Recorrências (9)

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Exercício 114

$$T^-(n) = \begin{cases} 0, & \text{se } n < 2, \\ 2T^-(\lfloor \frac{n}{2} \rfloor) + n - 1, & \text{se } n \geq 2 \end{cases}$$

Exercicio 114: $2T^-\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + n - 1$, se $n \geq 2$

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T. 29

$$T^-(n) = T^-(h^u(n)) \prod_{i=0}^{u-1} m(h^i(n)) + \sum_{i=0}^{u-1} s(h^i(n)) \prod_{j=0}^{i-1} m(h^j(n)), \text{ para todo } n \geq n_0,$$

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$$h(n) = \left\lfloor \frac{n}{2} \right\rfloor$$

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$$\begin{aligned} h(n) &= \left\lfloor \frac{n}{2} \right\rfloor \\ h^k(n) &= \left\lfloor \frac{n}{2^k} \right\rfloor \end{aligned}$$

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$$h(n) = \left\lfloor \frac{n}{2} \right\rfloor$$

$$h^k(n) = \left\lfloor \frac{n}{2^k} \right\rfloor$$

$$m(n) = 2$$

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$$h(n) = \left\lfloor \frac{n}{2} \right\rfloor$$

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$$m(n) = 2$$

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$$h(n) = \left\lfloor \frac{n}{2} \right\rfloor$$

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$$h(n) = \left\lfloor \frac{n}{2} \right\rfloor$$

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$$m(n) = 2$$

$$s(n) = n - 1$$

$$n_0 = 2$$

$$u = \min \left\{ k \in \mathbb{N} \mid h^k(n) < n_0 \right\}$$

Exercicio 114

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$$h(n) = \lfloor \frac{n}{2} \rfloor$$

$$h^k(n) = \lfloor \frac{n}{2^k} \rfloor$$

$$m(n) = 2$$

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Exercício 114

$$h(n) = \lfloor \frac{n}{2} \rfloor$$

$$h^k(n) = \lfloor \frac{n}{2^k} \rfloor$$

$$m(n) = 2$$

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$$T^-(n) = T^-(h^u(n)) \prod_{i=0}^{u-1} m(h^i(n)) + \sum_{i=0}^{u-1} s(h^i(n)) \prod_{j=0}^{i-1} m(h^j(n))$$

Exercicio 114

$$h(n) = \lfloor \frac{n}{2} \rfloor \qquad h^k(n) = \lfloor \frac{n}{2^k} \rfloor \qquad m(n) = 2 \qquad s(n) = n - 1 \qquad n_0 = 2$$

$$\begin{aligned} T^-(n) &= T^-(h^u(n)) \prod_{i=0}^{u-1} m(h^i(n)) + \sum_{i=0}^{u-1} s(h^i(n)) \prod_{j=0}^{i-1} m(h^j(n)) \\ &= T^-\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) \prod_{i=0}^{u-1} 2 + \sum_{i=0}^{u-1} s\left(\left\lfloor \frac{n}{2^i} \right\rfloor\right) \prod_{j=0}^{i-1} 2 \end{aligned}$$

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$$\begin{aligned} T^-(n) &= T^-(h^u(n)) \prod_{i=0}^{u-1} m(h^i(n)) + \sum_{i=0}^{u-1} s(h^i(n)) \prod_{j=0}^{i-1} m(h^j(n)) \\ &= T^-\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) \prod_{i=0}^{u-1} 2 + \sum_{i=0}^{u-1} s\left(\left\lfloor \frac{n}{2^i} \right\rfloor\right) \prod_{j=0}^{i-1} 2 \\ &= T^-\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) 2^u + \sum_{i=0}^{u-1} \left(\left\lfloor \frac{n}{2^i} \right\rfloor - 1\right) 2^i \end{aligned}$$

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$$T^-(n) = T^-\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) 2^u + \sum_{i=0}^{u-1} \left(\left\lfloor \frac{n}{2^i} \right\rfloor - 1\right) 2^i$$

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$$\begin{aligned} T^-(n) &= T^-\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) 2^u + \sum_{i=0}^{u-1} \left(\left\lfloor \frac{n}{2^i} \right\rfloor - 1\right) 2^i \\ &= 2^u T^-\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) + \sum_{i=0}^{u-1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - \sum_{i=0}^{u-1} 2^i \end{aligned}$$

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$$\begin{aligned}T^{-}(n) &= T^{-}\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) 2^u + \sum_{i=0}^{u-1} \left(\left\lfloor \frac{n}{2^i} \right\rfloor - 1\right) 2^i \\&= 2^u T^{-}\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) + \sum_{i=0}^{u-1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - \sum_{i=0}^{u-1} 2^i \\&= 2^u T^{-}\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) + \sum_{i=0}^{u-1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - (2^u - 1)\end{aligned}$$

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$$\begin{aligned}T^{-}(n) &= T^{-}\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) 2^u + \sum_{i=0}^{u-1} \left(\left\lfloor \frac{n}{2^i} \right\rfloor - 1\right) 2^i \\&= 2^u T^{-}\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) + \sum_{i=0}^{u-1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - \sum_{i=0}^{u-1} 2^i \\&= 2^u T^{-}\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) + \sum_{i=0}^{u-1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - (2^u - 1) \\&= 2^u T^{-}\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) + \sum_{i=0}^{u-1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - 2^u + 1\end{aligned}$$

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$$u = \min \left\{ k \in \mathbb{N} \mid \left\lfloor \frac{n}{2^k} \right\rfloor < 2 \right\}$$

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$$T^-(n) = 2^u T^-\left(\left\lfloor \frac{n}{2^u} \right\rfloor\right) + \sum_{i=0}^{u-1} 2^i \left\lfloor \frac{n}{2^i} \right\rfloor - 2^u + 1$$

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$$u = \min \left\{ k \in \mathbb{N} \mid \left\lfloor \frac{n}{2^k} \right\rfloor < 2 \right\} = \dots = \lfloor \lg n \rfloor$$

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$$u = \min \left\{ k \in \mathbb{N} \mid \left\lfloor \frac{n}{2^k} \right\rfloor < 2 \right\} = \dots = \lfloor \lg n \rfloor$$

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