Exercícios — Métodos Numéricos

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26 de Abril de 2024

1 Sistemas Lineares

1. Resolva o sistema abaixo usando: (a) Regra de Cramer; (b) Método de Gauss; (c) Método de Gauss com pivotamento parcial; e (d) Método de Jordan.

$$\begin{aligned} x_1 &- 3x_2 &+ 4x_3 &= 5 \\ -x_1 &+ 4x_2 &- 3x_3 &= 0 \\ 3x_1 &- 5x_2 &+ 2x_3 &= -7 \end{aligned}$$

$x_1 = -1; x_2 = 2; x_3 = 3$

2. Resolva o sistema abaixo usando o Método de Gauss-Jacobi (a) na forma matricial e (b) na forma algébrica, considere o valor inicial
$X_0 = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$ e o erro $\leq 0,05$.

$$\begin{aligned} 5x_1 &- x_2 &= 13 \\ 2x_1 &+ 4x_2 &= 14 \end{aligned}$$

i	x_1	$\lvert x_1^{(k)} - x_1^{k-1} \rvert$	x_2	$\lvert x_2^{(k)} - x_2^{k-1} \rvert$
1	**2.8**	1.8	**3.0**	2.0
2	**3.2**	0.4	**2.1**	0.9
3	**3.02**	0.18	**1.9**	0.2
4	**2.98**	0.04	**1.99**	0.09
5	**2.998**	0.018	**2.01**	0.02

3. Resolva o sistema abaixo usando o Método de Gauss-Jacobi, considere o valor inicial $X_0 = \begin{bmatrix} 1 & 1,5 & 2 \end{bmatrix}^T$ e o erro $\leq 0,05$.

$$\begin{array}{rcrcrcr} 3x_1 & + & 2x_2 & - & x_3 & = & 8 \\ 2x_1 & - & 4x_2 & + & 2x_3 & = & -4 \\ -x_1 & + & x_2 & + & 5x_3 & = & 3 \end{array}$$

i	x_1	$\lvert x_1^{(k)} - x_1^{k-1} \rvert$	x_2	$\lvert x_2^{(k)} - x_2^{k-1} \rvert$	x_3	$\lvert x_3^{(k)} - x_3^{k-1} \rvert$
1	**2.33333**	1.33333	**2.5**	1.0	**0.5**	1.5
2	**1.16667**	1.16667	**2.41667**	0.08333	**0.56667**	0.06667
3	**1.24444**	0.07778	**1.86667**	0.55	**0.35**	0.21667
4	**1.53889**	0.29444	**1.79722**	0.06944	**0.47556**	0.12556
5	**1.62704**	0.08815	**2.00722**	0.21	**0.54833**	0.07278
6	**1.5113**	0.11574	**2.08769**	0.08046	**0.52396**	0.02437
7	**1.44953**	0.06177	**2.01763**	0.07006	**0.48472**	0.03924
8	**1.48315**	0.03362	**1.96713**	0.0505	**0.48638**	0.00166
9	**1.51738**	0.03422	**1.98477**	0.01764	**0.50321**	0.01683

4. Resolva o sistema abaixo usando o Método de Gauss-Seidel (a) na forma matricial e (b) na forma algébrica, considere o valor inicial $X_0 = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$ e o erro $\leq 0,05$.

$$\begin{array}{rcrcr} 5x_1 & - & x_2 & = & 13 \\ 2x_1 & + & 4x_2 & = & 14 \end{array}$$

i	x_1	$\lvert x_1^{(k)} - x_1^{k-1} \rvert$	x_2	$\lvert x_2^{(k)} - x_2^{k-1} \rvert$
1	**2.8**	1.8	**2.1**	1.1
2	**3.02**	0.22	**1.99**	0.11
3	**2.998**	0.022	**2.001**	0.011

5. Resolva o sistema abaixo usando o Método de Gauss-Seidel, considere o valor inicial $X_0 = \begin{bmatrix} 1 & 1,5 & 2 \end{bmatrix}^T$ e o erro $\leq 0,05$.

$$\begin{array}{rcrcrcr} 3x_1 & + & 2x_2 & - & x_3 & = & 8 \\ 2x_1 & - & 4x_2 & + & 2x_3 & = & -4 \\ -x_1 & + & x_2 & + & 5x_3 & = & 3 \end{array}$$

i	x_1	$\lvert x_1^{(k)} - x_1^{k-1} \rvert$	x_2	$\lvert x_2^{(k)} - x_2^{k-1} \rvert$	x_3	$\lvert x_3^{(k)} - x_3^{k-1} \rvert$
1	**2.33333**	1.33333	**3.16667**	1.66667	**0.43333**	1.56667
2	**0.7**	1.63333	**1.56667**	1.6	**0.42667**	0.00667
3	**1.76444**	1.06444	**2.09556**	0.52889	**0.53378**	0.10711
4	**1.44756**	0.31689	**1.99067**	0.10489	**0.49138**	0.0424
5	**1.50335**	0.05579	**1.99736**	0.0067	**0.5012**	0.00982
6	**1.50216**	0.00119	**2.00168**	0.00431	**0.5001**	0.0011

6. Resolva o sistema abaixo usando o Método de Relaxação, considere o valor inicial $X_0 = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$ e o erro $\leq 0,05$.

$$\begin{array}{rcrcr} 5x_1 & - & x_2 & = & 13 \\ 2x_1 & + & 4x_2 & = & 14 \end{array}$$

i	x_1	x_2	R_1	R_2
1	1.0	1.0	−9.0	−8.0
2	2.8	1.0	0.0	−4.4
3	2.8	2.1	−1.1	0.0
4	3.02	2.1	0.0	0.44
5	3.02	1.99	0.11	0.0
6	2.998	1.99	0.0	−0.044

7. Resolva o sistema abaixo usando o Método de Relaxação, considere o valor inicial $X_0 = \begin{bmatrix} 1 & 1,5 & 2 \end{bmatrix}^T$ e o erro $\leq 0,1$.

$$\begin{array}{rcrcrcr} 3x_1 & + & 2x_2 & - & x_3 & = & 8 \\ 2x_1 & - & 4x_2 & + & 2x_3 & = & -4 \\ -x_1 & + & x_2 & + & 5x_3 & = & 3 \end{array}$$

i	x_1	x_2	x_3	R_1	R_2	R_3
1	1.0	1.5	2.0	−4.0	4.0	7.5
2	1.0	1.5	0.5	−2.5	1.0	0.0
3	1.83333	1.5	0.5	0.0	2.66667	−0.83333
4	1.83333	2.16667	0.5	1.33333	0.0	−0.16667
5	1.38889	2.16667	0.5	0.0	−0.88889	0.27778
6	1.38889	1.94444	0.5	−0.44444	0.0	0.05556
7	1.53704	1.94444	0.5	0.0	0.2963	−0.09259
8	1.53704	2.01852	0.5	0.14815	0.0	−0.01852
9	1.48765	2.01852	0.5	0.0	−0.09877	0.03086